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Acoustic multi-stopband metamaterial plates design for broadband elastic wave absorption and vibration suppression

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ABSTRACT

This paper presents the modeling technique, working mechanism and design guidelines for acoustic multi-stopband metamaterial plates for broadband elastic wave absorption and vibration suppression. The metamaterial plate is designed by integrating two-degree of freedom (DOF) mass–spring subsystems with an isotropic plate to act as vibration absorbers. For an infinite metamaterial plate without damping, a working unit is modeled using the extended Hamilton's principle, and two stopbands are observed through dispersion analysis on the averaged three-DOF model. For a finite metamaterial plate with boundary conditions and damping, shear-deformable conforming plate elements are used to model the whole plate, and stopbands are investigated by frequency response analysis and transient analysis. Influences of absorbers' resonant frequencies and damping ratios, plate's boundary conditions and dimensions, and effective plate-absorber vibration modes are thoroughly investigated. Results show that the metamaterial plate is essentially based on the concept of conventional vibration absorbers. The local resonance of the two-DOF subsystems generates two stopbands, and the inertial forces generated by the resonant vibrations of absorbers straighten the plate and attenuate/stop wave propagation. Each stopband's bandwidth can be increased by increasing the absorber mass and/or reducing the average mass of isotropic plate in each working unit. Moreover, while a low damping ratio for the primary absorber can guarantee absorbers' quick response to transient excitations, a high damping ratio for the secondary absorber can combine the two stopbands into a wide one. In the end, a sensitivity analysis on absorber resonant frequencies is conducted and relatively large sensitivity is found at the two stopband regions.

1. Introduction

The concept of metamaterials was first discussed in 1968 [1] but most of the early studies focused on electromagnetic metamaterials (EMs), which are materials with negative permittivity and permeability. Famous properties of EMs include negative refractive indices [2], ability of invisibility [3,4], and inverse Doppler effect [5]. Based on the similarity between electromagnetic waves and acoustic waves, a new type of metamaterials called acoustic metamaterials was proposed and investigated in recent years [6–10]. Popular research topics about acoustic metamaterials include ultrasound focusing [11], acoustic cloaking [12], elastic wave absorption [13] and structural vibration mitigation [14]. Because earthquake often generates destructive body and surface waves [15], seismic waveguides are an important application of acoustic metamaterials. For example, Kim and Das proposed a novel seismic attenuator made of metamaterials based on the characteristics of different seismic waves [16], and Brule et al. [17] experimentally investigated seismic metamaterials interaction with seismic waves by molding the surface waves.

Phononic crystals (PCs), sometimes also classified as a special kind of acoustic metamaterials by some researchers [18,19], have been under investigation since the 1990s [20–22]. Similar to acoustic metamaterials, PCs usually exhibit anomaly properties, such as stopbands (or bandgaps) [23], negative refractive index [24] and Fano profiles [25], but the difference between PCs and acoustic metamaterials is also significant. Detailed comparison between acoustic metamaterials and PCs have been done by lots of researchers [6,18,19]. PCs, analogous to the idea of photonic crystals, are artificial composite materials consisting of acoustic functional scatters of high impedance and matrix of low impedance. Because PCs are based on the idea of *Bragg scattering*, the scatters must be arranged spatially on the order of the matrix acoustic wavelength [6]. However, wavelengths of environmental low-frequency sound waves are usually large and hence absorption of such waves requires large size PCs, which usually limits the application [26]. One solution to this problem is to use acoustic metamaterials.

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Instead of relying on Bragg scattering effect, acoustic metamaterials utilize local resonance of inclusions, and hence the size of acoustic metamaterials can be far less than the wavelength. For example, the acoustic metamaterials proposed in 2000, consisting of rubber-coated lead balls, are two orders smaller than the incident wavelength [27]. Acoustic metamaterials can be further classified into *intrinsic* and *inertial* ones [6]. In intrinsic acoustic metamaterials, the phase speed of the inclusions (e.g., soft silicon rubber [28]) is much lower than that of the matrix [29,30]. Instead of requiring inclusions with low phase velocity, inertial acoustic metamaterials employ mass–spring–damper subsystems as local resonators. The inertial forces of the subsystems under resonance work against the excitation and attenuate the vibration. Early studies on inertial acoustic metamaterials focused on dispersion analysis and band structures of mass–spring lattice systems. For example, In 2003 phononic stopbands of 1-D and 2-D mass–spring lattice structures with two types of working units were extensively investigated [31]. Subsequently, different types of mechanical lattice structures with stopbands were proposed [32]. The effect of attaching mass–spring subsystems to a rigid body was investigated in 2007 [33]. In 2008 the negative effective mass and stopbands of a 1D mass–spring system was experimentally verified [34]. The experiment was conducted on an air track and CCD cameras were used to capture the motion of the masses.

Although inertial acoustic metamaterials with lattice structures have been widely discussed, metamaterials with continuum structures like bars, beams, plates and shells, which are more commonly used in engineering designs, have not been fully studied. In 2008 Cheng et al. [35] proposed a 1-D ultrasonic metamaterial beam showing simultaneously negative dynamic density and modulus. The metamaterial beam was constructed by attaching Helmholtz resonators to an elastic beam, and analyzed with an acoustic transmission line method. The results was confirmed by finite-element analysis using solid elements for both the elastic beam and the Helmholtz resonators. The model was latter improved with parallel-coupled Helmholtz resonators [36]. In 2008 and 2010 Wu [37] and Oudich [38] proposed metamaterial plates with periodic stubbed surface. Both of them reported stopbands in metamaterials either using single-material stubs with height three times the plate thickness, or using rubber stubs with metal caps. No matter the metamaterials are constructed by rubber-coated lead balls [27], Helmholtz resonators [35,36], or plates with stubbed surface [37,38], all the local resonators can be modeled by simple mass–spring subsystems. However, due to difficulties in modeling local deformation/motion of discrete mass–spring subsystems attached to continuum bodies by classical continuum theories, such models have not been well analyzed [39]. Zhu et al. [39] proposed a microstructure continuum model to tackle the problem but the displacement in the continuum body was approximated by linear series expansions in terms of quantities defined at the cell center, rather than modeled with classical continuum theory. A review of previous elastic metamaterial plates have been done by Zhu et al. [40].

Based on the classical continuum theory [41], we have done extensive work on metamaterial bars, beams and plates by modeling the discrete local resonators as essential mass–spring subsystems [7–10]. In 2010 a metamaterial bar made of a hollow longitudinal bar with mass–spring subsystems attached inside was introduced [7]. Dispersion analysis and finite-element modeling showed that a stopband was created by the subsystems, and the stopband was tunable by changing the resonant frequencies of the subsystems. Following a similar approach, a metamaterial beam was designed by attaching translational and rotational subsystems to an elastic beam [9]. Timoshenko's beam theory and rotary inertias were included in the model because shear deformation and rotary inertias are important for thick beams and/or high-frequency vibrations. Finite-element analysis showed a tunable stopband and the attached rotational inertias were proved to be not as efficient as the translational inertias. In 2014 the stopband is largely expanded by introducing a multi-stopband metamaterial beams. Two stopband are connected into a wide one by attaching a secondary vibration absorber to the primary one and applying large damping to the secondary vibration absorber [8]. Recently Peng and Pai [10] proposed metamaterial plates with mass–spring subsystems for wave absorption and vibration suppression.

Metamaterial plates have more engineering applications than metamaterial bars or beams. For example, metamaterial plates can be used to protect important building structures (e.g., museums, dams and school buildings) during earthquakes and reduce noise in residential halls. To the authors' knowledge, metamaterial plate models based on the idea of multi-frequency mass–spring absorbers have never been presented in the literatures. This paper is aimed to design a multi-stopband metamaterial plate for broadband vibration absorption by employing the local resonance between the multi-frequency absorbers and the external excitation. Each absorber consists of a primary and a secondary absorber and hence the multiple resonant frequencies of the subsystems can be used to attenuate/stop broadband elastic waves. Guidelines for appropriate design of primary and secondary absorbers are derived. Influences of absorbers' resonant frequencies and damping ratios, plate's boundary conditions and dimensions, and acoustic and optical plate-absorber vibration modes are fully investigated.

2. Multi-frequency vibration absorber

A single stopband exists right above the resonant frequency of a conventional single-mass vibration absorber [7,42]. Increase of the absorber damping can widen the stopband to some degree, but the effect is minimal. Moreover, large absorber damping slows down the absorber's response to an excitation and increases the transient time. These shortcomings of conventional vibration absorbers prompt the idea of using multi-frequency vibration absorbers. Fig. 1 shows a multi-frequency vibration absorber. The base system m is connected to the ground by the spring k and the damper c and subjected to a harmonic excitation f with an amplitude f_0 and a frequency ω. The displacement of the base system is denoted by u. Different from the conventional vibration absorber, two massesm_1 and m_2, instead of a single mass, are attached to the base system. The displacements of the primary and secondary masses are denoted by u_1 and u_2, respectively. This multiple-frequency vibration absorber is designed such that the

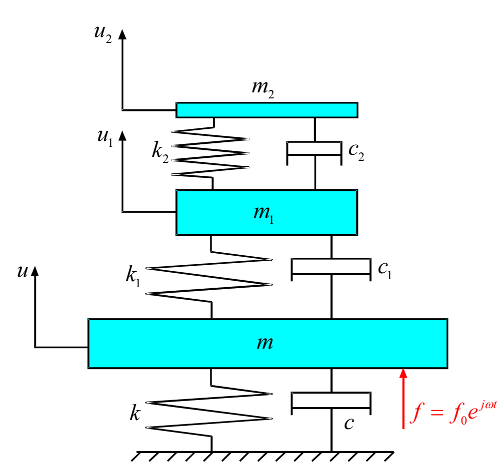

Fig. 1. A multi-frequency vibration absorber.