International Journal of Mechanical Sciences 103 (2015) 172–180

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Shakedown analysis of truss structures with nonlinear kinematic hardening

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ARTICLE INFO

Article history:
Received 20 September 2014
Received in revised form
7 September 2015
Accepted 10 September 2015
Available online 24 September 2015

Keywords:
Shakedown analysis
Truss structure
Hölder inequality
Duality

ABSTRACT

The paper aims to perform shakedown analysis of truss structures with nonlinear kinematic hardening. Both the static and kinematic shakedown analyses of truss structures were conducted analytically and numerically to bound the shakedown limit. First, the problem statement leads to the lower bound (primal) formulation accounting for nonlinear kinematic hardening extended from the Melan's static shakedown theorem. In particular, the Hölder inequality is then utilized to establish the corresponding upper bound (dual) formulation from the lower bound formulation as well as to confirm the duality relationship between them. The derived upper bound formulation is an equivalent form of the Koiter's kinematic shakedown formulation for trusses without involving time integrals. Further, both the primal and the dual analyses of truss structures were conducted using the optimization algorithms provided by MATLAB. Accordingly, the primal analysis and the dual analysis are validated by converging to the shakedown limit efficiently. Finally, the step-by-step finite-element analysis by using ABAQUS is also performed to verify the analytical formulation and numerical implementation.

1. Introduction

Engineering structures are often subjected to cyclic loads. Structures of elastic–plastic materials under cyclic loads may behave in some different types, namely pure elastic, elastic shakedown, ratcheting (incremental collapse), plastic shakedown (reversed plasticity) and plastic collapse (unconstrained plastic flow) [1,2]. Among them, elastic shakedown, ratcheting and plastic shakedown are three types of elastic–plastic responses induced by the cyclic loads ranging between the elastic limit and plastic collapse loads. An elastic–plastic structure is said to shake down to an elastic state if it deforms plastically in the initial loading cycles and then reacts purely elastically in the sequential cycles e.g. [1–5]. As well known, shakedown analysis is a well-established direct method to determine the shakedown limit based on the lower bound (Melan's static) or upper bound (Koiter's kinematic) theorem e.g. [1–5]. By the shakedown theorems, we can formulate shakedown analysis into constrained optimization problems by mathematical programming techniques e.g. [6] to seek the shakedown limit. On the one hand, we seek the greatest lower bound by the static theorem e.g. [7,8]. On the other hand, we search for the least upper bound by the kinematic theorem e.g. [9,10]. Moreover, the duality holding between the lower and the upper bound formulations has been well revealed in shakedown analysis e.g. [11–22]. Accordingly, we can apply the lower and upper bound theorems to bound the exact shakedown limit from below and above, respectively.

The Melan's static shakedown theorem was originally stated for structures made of elastic-perfectly plastic materials. However, real-life materials generally demonstrate kinematic hardening behavior [23]. Accordingly, it is more realistic to take kinematical hardening into account while dealing with shakedown analysis problems. In literature, much effort has been made to extend the classical shakedown theorems to consider the effects of hardening [24–29]. Compared to other hardening models, the Armstrong–Frederick nonlinear kinematic hardening model [30,31] is more realistic one for shakedown analysis of metals. However, it seems that no effort has been made to shakedown analyses of truss structures with nonlinear kinematic hardening. The paper aims to perform shakedown analysis of truss structures with nonlinear kinematic hardening. It may be formidable to derive the corresponding shakedown limits directly. However, it is possible to acquire the exact solutions by the duality between static and kinematic formulations. Namely, we can approach from below and above the shakedown limit by conducting static and kinematic shakedown analyses of truss structures, respectively. In the paper, we first state the problem statement of shakedown analysis in the form of the lower bound (primal) formulation. The lower bound

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http://dx.doi.org/10.1016/j.ijmecsci.2015.09.003
0020-7403/

(primal) formulation is then transformed to the upper bound (dual) formulation.

On the other hand, limit analysis is a special case of shakedown analysis. Accordingly, there exist many similarities between limit and shakedown analysis. In limit analysis, Yang [32] applied the Hölder inequality [33] to establish the kinematic (dual) formulation transformed from the corresponding static (primal) formulation. In particular, Yang [32] stated the primal (lower bound) formulation with the yield criterion denoted in the form of l_∞-norm while dealing with limit analysis of truss structures. Yang [32] applied the Hölder inequality [33] to establish the corresponding dual (upper bound) formulation with the l_1-norm on plastic deformation rate of truss members.

Following the successful experience in limit analysis of truss structures [32], the paper aims to perform shakedown analysis of truss structures with nonlinear kinematic hardening. Both the static and kinematic shakedown analyses of truss structures were conducted to bound the exact shakedown limit. First, the problem statement of shakedown analysis is to be stated in the form of the lower bound (primal) formulation involving with l_∞-norm of axial forces [32]. By the Hölder inequality [33], the lower bound (primal) formulation is then transformed to the upper bound (dual) formulation with the l_1-norm [32]. The equality relationship between the greatest lower bound and the least upper bound is to be analytically confirmed to illustrate the strong duality between the lower and upper bound formulations. To illustrate numerically the strong duality between the lower and upper bound formulations, the primal and the dual analysis are to be performed by the computing tool MATLAB [34], respectively. Finally, the step-by-step finite-element analysis by using ABAQUS is also performed to verify the analytical formulation and numerical implementation.

2. Analytical background

We consider truss members made of materials with nonlinear kinematic hardening. The Armstrong–Frederick kinematic hardening model [30,31] is adopted. Corresponding to the Armstrong–Frederick nonlinear kinematic hardening for a von Mises material, the yield function is denoted as [35]

$$f(\sigma - X) = \sqrt{\frac{3}{2}(S - X^{dev}) : (S - X^{dev})} - \sigma_0 \tag{1}$$

where S is the deviatoric stress tensor, X^{dev} is the deviatoric part of the backstress tensor X acting to translate the center of the yield surface, σ_0 is the yield strength. It is noted that the backstress X denotes the movement of the yield surface center. Accordingly, the convexity of the yield surface preserves for a von Mises material with nonlinear kinematic hardening.

By the Armstrong–Frederick kinematic hardening model [30,31], the backstress rate $\dot{X}$ is described as

$$\dot{X} = \frac{2}{3}C\dot{\varepsilon}^p - \gamma X \dot{\bar{\varepsilon}}^p \tag{2}$$

where C and γ are material parameters, $\dot{\varepsilon}^p$ is the plastic strain rate, $\dot{\bar{\varepsilon}}^p$ denotes the equivalent plastic strain rate.

As well known, truss members carry only axial forces. For uniaxial loading in the 1-direction, we have stress tensors $\sigma_{ij} = 0$ except $\sigma_{11} \neq 0$. By the incompressibility condition and symmetry, we have the strain rate tensors $\dot{\varepsilon}^p_{22} = \dot{\varepsilon}^p_{33} = -\dot{\varepsilon}^p_{11}/2$. On the other hand, the backstress is also a deviatoric tensor with $X_{22} = X_{33} = -X_{11}/2$[35].

Furthermore, we have the equivalent stress $\bar{\sigma}$ associated with the von Mises yield criterion expressed as

$$\begin{aligned}\bar{\sigma} &= \sqrt{\frac{1}{2}[(\sigma_{11}-X_{11})-(\sigma_{22}-X_{22})]^2 + \frac{1}{2}[(\sigma_{22}-X_{22})-(\sigma_{33}-X_{33})]^2 + \frac{1}{2}[(\sigma_{33}-X_{33})-(\sigma_{11}-X_{11})]^2} \\ &= |(\sigma_{11}-X_{11})-(\sigma_{22}-X_{22})| \\ &= \left|\sigma_{11} - \frac{3}{2}X_{11}\right|\end{aligned} \tag{3}$$

Thus, the yield function can be simplified as [36]

$$f = \left|\sigma_{11} - \frac{3}{2}X_{11}\right| - \sigma_0 \tag{4}$$

Due to the uniaxial loading condition, it is convenient to describe the yield behavior of truss members in terms of axial forces. Thus, the yield condition for the $i-$th truss member with initial cross sectional area A_0 and yield strength σ_0 can be generally described as

$$A_0\left|\sigma^{(i)}_{11} - \frac{3}{2}X^{(i)}_{11}\right| = \left|t^{(i)} - \frac{3}{2}x^{(i)}\right| \leq A_0\sigma_0 \tag{5}$$

where $t^{(i)} = A_0\sigma^{(i)}_{11}$, $x^{(i)} = A_0 X^{(i)}_{11}$ are the axial force and the (force-like) axial backstress of the $i-$th truss member, $\sigma^{(i)}_{11}$ and $X^{(i)}_{11}$ are the corresponding axial stress and axial backstress, respectively.

Note that, if we normalize the constitutive model as follows

$$\left|\frac{t^{(i)}}{A_0\sigma_0} - \frac{3x^{(i)}}{2A_0\sigma_0}\right| = \left|\bar{t}^{(i)} - \frac{3}{2}\bar{x}^{(i)}\right| \leq 1 \tag{6}$$

with

$$\bar{t}^{(i)} = \frac{t^{(i)}}{A_0\sigma_0} = \frac{\sigma^{(i)}_{11}}{\sigma_0} \tag{7}$$

$$\bar{x}^{(i)} = \frac{x^{(i)}}{A_0\sigma_0} = \frac{X^{(i)}_{11}}{\sigma_0} \tag{8}$$

Then we can state the constitutive model in the form of l_∞-norm as

$$\left\|\bar{\mathbf{t}} - \frac{3}{2}\bar{\mathbf{x}}\right\|_\infty = \max_i \left\{\left|\bar{t}^{(i)} - \frac{3}{2}\bar{x}^{(i)}\right|\right\} \leq 1 \tag{9}$$

where $\bar{\mathbf{t}}$, $\bar{\mathbf{x}}$ are vectors with components $\bar{t}^{(i)}$, $\bar{x}^{(i)}$, respectively.

On the other hand, the equivalent plastic strain rate $\dot{\bar{\varepsilon}}^p$ associated with the von Mises yield criterion is expressed as

$$\begin{aligned}\dot{\bar{\varepsilon}}^p &= \sqrt{\frac{2}{9}\left[(\dot{\varepsilon}^p_{11}-\dot{\varepsilon}^p_{22})^2 + (\dot{\varepsilon}^p_{22}-\dot{\varepsilon}^p_{33})^2 + (\dot{\varepsilon}^p_{33}-\dot{\varepsilon}^p_{11})^2\right]} \\ &= \sqrt{\frac{4}{9}\left(\frac{3}{2}\dot{\varepsilon}^p_{11}\right)^2} \\ &= |\dot{\varepsilon}^p_{11}|\end{aligned} \tag{10}$$

For uniaxial loading with the initial condition $X(0) = 0$, the values of the backstress X_{11} can be obtained by Eq. (2) as follows

For uniaxial tension, we have [37]

$$X^{(i)}_{11} = \frac{2}{3}\frac{C}{\gamma}\left(1 - e^{-\gamma\varepsilon^{p(i)}_{11}}\right) \tag{11}$$

where $\varepsilon^{p(i)}_{11}$ is the axial plastic strain of the $i-$th truss member.

For uniaxial compression, we have [37]

$$X^{(i)}_{11} = \frac{2}{3}\frac{C}{\gamma}\left(-1 + e^{\gamma\varepsilon^{p(i)}_{11}}\right) \tag{12}$$

Thus, the backstress of the $i-$th truss member, $X^{(i)}_{11}$, will be expressed as Eqs. (11) or (12) depending on the truss member subjected to tensile or compressive loading.