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Attractor reconstruction of an impact oscillator for parameter identification

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ABSTRACT

This paper presents a simple and novel approach, based on nonlinear time series analysis of an experimental system, to infer from subtle alteration of the system dynamics the changes caused in the system parameters. Using the acceleration time-series as a measurement of simulated and experimental impact oscillators (serving as a model for the drilling conditions with intermittent contact between the drill- bit and the formation), the systems attractor is reconstructed and characterised. It is shown that the stiffness correlates with the topology of the reconstructed attractor. Non-impacting trajectories form an approximate plane within the three dimensional reconstructed phase-space, and contact with the constraint causes a systematic deviation from the linear subspace, the inclination of which, measured by the statistics of the tangent vector, can be used to infer the stiffness. This relationship between the topology of attractor and the stiffness was also verified experimentally. Based on the developed framework, it is now possible to classify the stiffness of the impacted material from a single variable in a simple way and in real time.

1. Introduction

Systems with impacts have received a great deal of attention both in the study of fundamental non-smooth dynamics and in engineering applications. Periodic impacts of colliding bodies in mechanical systems such as gears can be highly detrimental to their life span, and can generate vibration and noise. In contrast, some other mechanical systems rely on impact motion to perform their intended function. An example of such systems is percussive drilling, where higher impact performance is required to increase the penetration rate through hard rock formations. Hence, advanced understanding of impacting systems should help in prolonging their usable life span by controlling vibration and noise levels and/or to improve their performance and functionality.

Impacting systems have been studied extensively in the literature mainly assuming a harmonically forced oscillator having rigidly or flexibly constrained vibration amplitude. In both cases, the systems exhibit periodic and complex behaviour including chaos. Peterka and Vacik [1] studied numerically the transition to chaos in a system with impact. In [2] Peterka studied non-smooth characteristics arising from impact mechanical system with soft and preload stops. Půst and Peterka [3] analysed the dynamical properties of a single degree-of-freedom mechanical system with a soft restraint using Hertz contact theory. They emphasised that the applied model based on the Hertz contact law could describe the real behaviour of impact systems better than other models, because it respects the nonlinearity of the restoring force between the impacted bodies. System parameters were studied and the phase-space portraits showed as observed in the system studied in this work, an enlargement of the trajectories as impact stiffness was increased and the energy lost during the impacts is increased with increase of the impact velocity. In Di Bernardo et al. [4] a non-smooth dynamical systems theory was further developed.

Thompson and Ghaffari [5] studied an impact oscillator rebounding elastically whenever the relative displacement drops to zero. Chaos is shown to appear through period doubling cascades. Shaw and Holmes [6] studied a model of a periodically forced oscillator with a flexible constraint (on impact system with single discontinuity) and analysed its corresponding bifurcations. Later, Shaw [7,8] studied the impacting responses and their stability on a system having two sided amplitude constraints. Whiston [9] studied the steady state non-chaotic responses of a harmonically excited piecewise linear oscillator. Nordmark [10] took the analysis of impact system further and used it to study the response of a single degree-of-freedom oscillator with a hard impact. The results have been derived from a system with instantaneous impact. The author has utilised the phase-space

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geometrical representation of the system to study different impact conditions. Analytical methods were developed to construct one-dimensional map between the different phase-space sections to study the singularities and stability of the orbits caused by different impact conditions. A square root singularity in the Jacobian matrix was found to appear exactly at the grazing impact conditions.

Extensive studies have been carried-out at the Centre for Applied Dynamic Research (CADR), the University of Aberdeen, on the dynamical behaviour of the impact systems. Different sizes experimental rigs were used to study the behaviour of drill-string where the rotation and the axial excitation were combined to drill vertical and horizontal holes on real rock formations. Comprehensive studies had also been carried out by the CADR on a piecewise linear oscillator system impacting against elastic beam. The impact oscillator experimental rig was originally developed by Wiercigroch and Sin [11,12] to study bifurcations of a system with a two sided restraint. Later, Ing et al. [13] modified it by removing one constraint to examine experimentally the bifurcation scenarios of near grazing conditions. Their study was supported by a robust mathematical model developed for the same setup. Later, the same authors [14] conducted experimental and semi-analytical studies on the behaviour of the impact oscillator under different exciting parameters. Smooth and non-smooth bifurcations were observed experimentally and explained using mapping solutions. Pavlovskaia et al. [15] studied the behaviour of a linear oscillator constrained by an elastic beam. Several bifurcations for near grazing conditions were observed experimentally. The results showed that the attractor evolution is influenced by an interplay between smooth and non-smooth behaviour.

Even though there is an extensive literature on nonlinear analysis to characterise the behaviour of nonsmooth dynamical systems, there is a need for experimental methods to help determining the moment, the conditions, and the parameters of the impact and of the impacted material, topics that constitute the main purpose of this work. Nonlinear time series analysis will be applied in here to investigate the underlying properties of this impacting mechanical system.

There are several examples in the literature explaining the application of nonlinear time series analysis to study the behaviour of engineering systems. Craig et al. [16] applied phase-space reconstructing techniques and measured the fractal dimension to monitor a dynamical system with bearing clearance. Their system was representing an elastically supported rotor excited by imbalance and restricted to two dimensional movements. The study was conducted on three different magnitudes of clearance using two different shaft speeds. Initially, conventional embedding delay method [33] was used to reconstruct the phase-space for time series recorded in one direction, but it resulted in bad quality and unacceptable reconstructions leading to substandard computation of correlation dimension. This was reasoned to be a consequence of the weakening of the coupling between the two motions in the two dimensions. Also, increasing the magnitude of the clearance caused more information lost about each direction. A modified method to reconstruct the phase-spaces was then used by considering values from other observation recorded from the system into the reconstructing vector. The study showed that the computed correlation dimension decreases as the magnitude of clearance is increased. This was observed in each case of the shaft speed. The authors emphasised the importance of using correlation dimension techniques on monitoring events related to the mechanical system. The work in [16] shows an example in engineering where the choice of the variables may deeply affect the quality of the reconstructed phase-space [34]. Some variables or set of variables may be better to reconstruct the original dynamics. When dealing with experimental data, the choice of the best variable can be conducted by following the work in [35]. In this work, the mass acceleration was selected as a variable to reconstruct the attractor, mainly because it can be used to identify the exact time of impact. For a reasonably low level of noise, this variable corresponds to the second time derivative of the displacement. Since this variable is related to the forces involved in the system studied and consequently to all the relevant state variables, had calculated the observability matrix from it (x_1, x_2). We concluded that it is likely to be fully invertible, meaning that this variable can in fact lead to a fully reconstructed phase-space.

Da Silva et al. [17] discussed the application of the nonlinear time series analysis to study experimental data when drilling in a semi-insulating Gallium arsenide, GaAs sample. They as well as we, in this work, used the method of false nearest neighbours to estimate the embedding dimension of the attractor, the number of axes, which needed to be considered in the embedded space in order to obtain the true reconstructed attractor. The correlation dimension and maximum Lyapunov exponent were computed for the reconstructed phase-space to obtain a better understanding of the dynamics of the experimental system in order to develop models for numerical simulations. A noteworthy feature of their article is that all of the computations were performed using a special-purpose time series analysis software called TISEAN [18]. Reiss and Sandler [19] also applied nonlinear time series analysis to electronic signals generated by a synthesiser to observe harmonic structures over long time spans. The study concluded that the nonlinear time series analysis techniques have a tendency to identify predominantly short term dynamics. This is due to the effect of experimental noise creating local non-invertible observability (x_1, x_2), preventing one from having a system fully observable from the measured variable, imposing limitation in the ability of long term prediction. Piiroinen et al. [20] studied a single degree-of-freedom system representing horizontally excited pendulum restricted by a rigid stopper. The time series showed a sudden change to chaotic and period-adding cascade behaviours as a parameter is altered, which is the expected dynamics that could be predicted from a grazing bifurcation normal form. These results were also achieved from the numerical simulation of the system, where discontinuity map was applied to derive the coefficient of the square-root normal-form map. Numerical continuation method was also applied to find the grazing periodic orbit and the linearisation of the system along the orbit. Then the system parameters were computed, which allow changing in the system behaviour to chaos or period-adding cascade. The study also highlighted the important role of the damping coefficient in the observed dynamics, and the importance of using correct amplitude in order to get close quantitative agreement between the numerical and experimental results. This implies that the amplitude might require an adjustment to overcome the effects of the parameters that might not be considered in the model. Kodba et al. [21] have studied a signal of a RLC-diode circuit driven by an alternating voltage source of variable amplitude. Using systematically nonlinear time series analyses the author observed that as the strength of the voltage source is increased, the system reaches a chaotic state through a period doubling route. Perc [22] also applied the same techniques mentioned in the previous papers to study the dynamic of electrocardiogram and human gait systems [23]. The study emphasised that the time series recoded for a humangait possesses deterministic chaotic nature which was consistent with other related studies.

Detecting an impact between the mass and the restraint, and its stiffness from experimental data is not an easy task especially if the impact is near grazing and also if the noise level was high. The experimental setup may require making use of a more sophisticated sensing system, thus more complicated and sensitive devices would be needed. Ing [25] and Ing et al. [13] investigated the