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The analysis and model formulation of a coupled micro-probe and elastic thin plate subjected to electrostatic force

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ABSTRACT

The mathematical model of coupled probe-plate system subjected to the ac and dc voltages is constructed. Among that, the coupled interacting force between probe and plate is the electrostatic force due to the ac and dc voltages, and the coupled displacements of beam and plate occur simultaneously. It is different to the conventional micro-/nano-actuator which is constructed by two independent fixed/mobile conducting electrodes. It is worth noting that the pull-in phenomenon of the coupled system subjected to the dc voltage only is discovered as a significant difference with respect to the conventional one. In this study, the analytical method for the coupled vibration is presented; the suitability of the conventional perturbation method is investigated; the relationship between the coupled frequencies of the system and the frequencies of the probe and plate is also found. This relationship is defined as the coupled characteristic phenomenon here.

1. Introduction

Advances in electromechanical systems are resulting in new applications ranging from mechanical mass or charge detectors to biological imaging [23,35]. The pull-in phenomenon is widely applied in many micro-, nano- and quantum-machined actuators under electrostatic force [39,40]. Moreover, the investigation about the frequency shift is of the interest in designing sensors and actuators [24]. These can be classified into several kinds of structure subjected to electrostatic force: (1) single beam, (2) double or several beams, (3) plate, (4) beam-plate assembly. The relevant literatures are introduced as follows:

(1) *Single beam subjected to electrostatic force*:
  (a) *Static application*
  An assembly composed of a movable beam and a fixed electrode is usually used for the design of micro-, nano- and quantum-machined switches. The movable beam electrode deflects to the fixed electrode due to the electrostatic attraction caused by a voltage difference in between. The movable electrode becomes unstable and pull-in onto the ground electrode at a certain voltage. This critical voltage is called as a pull-in voltage of the switch. Zhang and Zhao [39] investigated the pull-in voltage and displacement of the individual beam or plate. It was demonstrated that the pull-in parameter which is the ratio of the dc voltage and the bending rigidity increases by increasing the axial load. Zhang et al. [40] produced a review about the electrostatic pull-in instability in MEMS/NEMS.
  (b) *Dynamic application*
  Nayfeh et al. [28] investigated the dynamic pull-in phenomenon in MEMS resonators. Hassanpoura et al. [11] investigated the influence of the concentrated mass on the natural frequency of a beam-type resonator. Kang et al. [13] investigated the ultrahigh frequency nano-resonators based on double-walled carbon nanotubes with different wall lengths. Prabhakar et al. [31] studied the frequency shifts due to thermoelastic coupling in flexural-mode single beam as a resonator. The Galerkin technique was used to calculate the thermoelastically shifted frequencies. Forke et al. [6] investigated the electrostatic force coupling of MEMS oscillators for spectral vibration measurements. In order to measure vibration, the sensor output signal was designed to be linearly dependent on the amplitude of acceleration. In other words, the electrostatic coupling force was $F_e = 0.5V^2 dC/dz$, where V is the voltage and the spatial derivative of the capacitance dC/dz, which was proportional to the distance z. Consequently, the comb electrodes were configured with linearly varying finger lengths but in a differential arrangement. Obviously, the investigations above are linear. As in these applications, the electromechanical system is also applied to the atomic force microscopy. The Kelvin probe

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Nomenclature

A_b	cross-sectional area of beam
A_H	Hamaker constant
D	tip-surface distance
D_d	distance between tip and plate, $D_0 - W_{bp}$
D_p	bending rigidity of plate, $Eh^3/[12(1-\mu^2)]$
E	Young's modulus
F_b	interacting force between beam and plate
f_b	dimensionless interacting force between beam and plate, $F_b L_b^2/E_b I_b$
g	acceleration of gravity
$\overline{g}$	ratio of weight and bending rigidity of plate, $\rho h L_1^3 g/D$
h	thickness of plate
H	height of tip
L_x,L_y	lengths of plate in the x- and y-directions, respectively
m_t	tip mass of beam
r_x, r_y	aspect ratios, L_x/L_b,L_y/L_b
r_{mass}	ratio of masses of plate and beam, $\rho_p h_p L_b/\rho_b A_b$
r_{rigid}	ratio of bending rigidities of beam and plate, $E_b I_b/D_p L_b$
R_0	radius of tip
s	coordinate of beam
t	time variable
V_{ac}	ac voltage
V_{dc}	dc voltage
V_0	small residual surface potential
W_b, W_p	transverse displacements of beam and plate
$\overline{W}_{bp}$	coupled amplitude of beam and plate, $\overline{W}_b(L_b) - \overline{W}_p(x_c, y_c)$
$\overline{w}_{bp}$	dimensionless coupled amplitude of beam and plate, $\overline{W}_{bp}/L_b$
w_b, w_p	dimensionless transverse displacements of beam and plate, $W_b/L_b, W_p/L_b$
x,y,z	principal frame coordinates of plate
x_c,y_c	interacting position
μ	Poisson's ratio
μ_{tip}	dimensionless tip mass of beam, $m_t/\rho_b A_b L_b$
ε_b	dimensionless principal frame coordinate, s/L_b
ε_0	vacuum permittivity
Ω	natural frequency
ω	dimensionless natural frequency,$\Omega L_b^2\sqrt{\rho_b A_b/E_b I_b}$
Ω_a	frequency of ac voltage
Ω_{mn}	natural frequency of plate
ξ_p, ζ_p	dimensionless principal frame coordinates of plate, x/L_b, y/L_b
ξ_c,ζ_c	dimensionless principal frame coordinates of moving mass, x_c/L_b, y_c/L_b
ρ	mass density
τ	dimensionless time, $\left(t/L_b^2\right)\sqrt{E_b I_b/\rho_b A_b}$
∇^2	Laplace's operator

Subscript

b	beam
p	plate

force microscopy subjected to the ac electrostatic force is currently used to image the protein and the contact potential difference on a large variety of samples, such as semiconductor and organic materials [18,19,22]. An et al. [1] emphasized the progress of application of Kelvin probe technique to electrochemical research in the past decade. Moores et al. [27] compared the resolution of frequency modulation (FM-KPFM), amplitude modulation (AM-KPFM), and lift modes KPFM for imaging the local electrical surface potential of complex biomolecular films and demonstrated that FM-KPFM mode had superior resolution for biological applications. Cook et al. [4] determined the local Volta potential differences between a platinum coated AFM tip and various pure metal specimens with a thin humidity induced surface electrolyte layer via scanning Kelvin probe force microscopy (SKPFM). They anticipated that SKPFM calibration will enhance the usefulness of this technique for atmospheric corrosion studies of metals under thin electrolyte layers and/or in the presence of ultra-thin humidity induced moisture layers. Berger et al. [2] applied the electrical modes in scanning probe microscopy to help a greater understanding of the electrical function of materials that were structured on the nanometer scale. They accentuated the use of the existing electrical modes was unique for the correlation of structural and electric information on a nanometer scale. Park et al. [30] demonstrated a novel approach based on KPFM imaging and measurement for ultra-sensitivity detection of nano-toxic silver ion using a single droplet of analytical solution. Szwajca et al. [35] investigated the characterization of self-assembled monolayers from aliphatic thiols with different chain length and termination on InAs (100) planar surfaces. Ellipsometry, contact angle measurements and atomic force microscopy (AFM) indicated the formation of smooth surface conforming monolayer. The literatures above are for a single beam subjected to electrostatic force.

(2) *Several-beams assembly subjected to electrostatic force*:
The double-beams assembly is broadly adopted in civil, mechanical, and aerospace engineering, such as cranes, resonators, spectrometers and interferometers. Some literatures are devoted in this field.

- (a) *Static application*
  Zhang et al. [38] investigated the buckling of a double-beam system under compressive axial loading. The two coupled beams were simply supported and continuously joined by a Winkler elastic layer. Zamanian and Karimiyan [37] investigated the mechanical behavior of a doubled micro-beam configuration under dc electrostatic actuation by Galerkin method and the commercial software, ANSYS.
- (b) *Dynamic application*
  Oniszczuk [29] investigated the forced vibration of an elastically connected simply supported double-beam system. Gao and Cheng [7] investigated the active vibration isolation of a two beams assembly with a piezoelectric actuator. De Rosa and Lippiello [5] investigated the free vibration of double-beams by using the differential quadrature method. Sadek et al. [33] presented the computational method for solving optimal control of a system of parallel beams. Li and Hua [17] studied the vibration of an elastically damped connected three-beam system. One thing in common is linear behavior in these investigated systems. On the contrary, Lin [23] investigated the mechanism of the double-beams assembly subjected to the nonlinear ac electrostatic force.

(3) *Plate subjected to electrostatic force*:

- (a) *Static application*:
  An assembly composed of a movable plate and a fixed electrode is also used for the micro-, nano- and quantum-machined switches. Zhang and Zhao [39] also investigated