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Size-dependent electro-elasto-mechanics of MEMS with initially curved deformable electrodes

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ABSTRACT

The aim of this paper is to examine the nonlinear size-dependent electro-elasto-statics/dynamics of a MEMS with an initially curved deformable electrode, based on the modified couple stress theory; in particular, the nonlinear motion characteristics of an initially curved deformable electrode actuated by a combination of DC and AC voltages are examined, taking into account small-size effects through use of the modified couple stress theory. The deformable electrode is modelled by means of an initially curved Euler–Bernoulli microbeam theory; the coupling between the electrical field and restoring microbeam force is demonstrated by electrical/displacement nonlinearities in the continuous model of the system, which is obtained by means of Hamilton's principle. The Galerkin method is employed to obtain a high-dimensional reduced-order model of the continuous system. The reduced-order model is solved by means of the pseudo-arclength continuation technique in order to analyse pull-in instabilities, snap-through motions, and nonlinear dynamical behaviour. Particularly, the electro-elasto-static deformation of the deformable electrode and pull-in instabilities are analysed when the system is subject to a DC voltage. The electro-elasto-dynamic motions of the deformable electrode are also analysed when the system is subject to both DC and AC voltages; the AC frequency-motion and AC amplitude-motion characteristics of the system are examined in the presence of coupled mechanical and electrical non-linearities. A stability analysis is conducted via the Floquet theory. The importance of employing the modified couple stress theory, rather than the classical continuum theory, is highlighted by comparing the system response based on each of these theories.

1. Introduction

1.1. Applications and fundamentals

Micro-electro-mechanical systems (MEMS) can be found in many engineering applications, e.g. in micro energy harvesters [1], accelerometers, temperature sensors, pressure sensors [2], switches [3,4], mass flow sensors, biosensors [5], and resonators [6]. Different actuation methods, such as piezoelectric, piezo-resistive, thermal, and electromagnetic are usually used; however, the electric actuation is the most widely employed (which consists of both DC and AC components), mainly due to its low power consumption. An electrically actuated MEMS mainly consists of two electrodes; first one is a deformable (movable) conductive electrode, and the second one is a rigid (stationary) conductive electrode – these two are separated by a dielectric medium (usually air). The electric force, applied on the deformable electrode due to the DC and AC voltages, is balanced by the elastic restoring force (due to the stored potential strain energy), and hence the deformable electrode starts oscillating. A fundamental question may be raised is that what if this balance is lost? Briefly, the answer is that when the electric potential overcomes the elastic potential, the deformable electrode collapses on the stationary one [7]; this type of collapse is called pull-in, more specifically, static pull-in, if the electrical potential is of a DC type, and dynamic pull-in, if the electrical field is of an AC type.

1.2. Initially curved deformable electrodes

Initially curved deformable electrodes are highly possible to be produced due to an improper manufacturing process. Apart from this, they are present in some MEMS applications, for instance in band-pass filters, microswitches, and microshutters. Hence, it is

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more practical to analyse the electro-elasto-mechanics of *initially curved* deformable electrodes rather than *perfectly straight* ones, as done in this paper.

1.3. Size-dependent deformation behaviour

One of interesting features of microscale elements, such as a deformable electrode, is that, due to their small sizes, a phenomenon known as the size-dependent deformation behaviour is displayed by them [8,9]. This experimental finding could not be predicted by the classical continuum mechanics; therefore, different theories, such as the modified couple stress and strain gradient theories, have been developed and employed in order to predict the size-dependent phenomenon theoretically [10,11]. The current paper utilises the modified couple stress theory in the theoretical modelling of an initially curved deformable electrode.

1.4. Literature review on both experiment and theory

Starting with the experimental investigations, Go et al. [3] presented the design, fabrication and testing of prestressed bimorph microbeams for applications in adjustable acceleration switches. Jin and Wang [12] fabricated an electrostatic resonator on a single-crystal silicon and investigated its second superharmonic resonant behaviour. Mestrom et al. [13] investigated the dynamic behaviour of MEMS resonators both theoretically and experimentally; they employed a single degree-of-freedom model for the theoretical part and compared the results to the experimental data obtained for the steady-state dynamic behaviour of the system. Li et al. [2] contributed to the field by examining the nonlinear dynamics of an electro-thermally excited pressure sensor employing both experimental and theoretical methods; for the theoretical part, they employed the method of multiple scales along with a single-mode Galerkin approximation in order to examine the response of the system.

There are many studies in the literature which examined the theoretical aspects of MEMS; first, the studies in which only the electro-elasto-static deformation is analysed, are reviewed. For instance, Mojahedi et al. [14] employed a perturbation technique in order to examine the static pull-in instability of electrostatically actuated microbeams. Baghani [15] contributed to the field by examining the response of an electro-statically actuated microcantilever analytically on the basis of the modified couple stress theory. Rahaeifard et al. [16] employed the same theory to investigate the size-dependent deflection and static pull-in of microbridges. The investigations were continued by Do et al. [17], who developed a generalised closed-form model for predicting the pull-in instability position and pull-in voltage of a microcantilever. Zhang and Zhao [18] studied the pull-in instability of electrostatically actuated microstructures via both numerical and analytical techniques, employing a single-degree-o-freedom system. Nayfeh and Younis [19] examined the nonlinear dynamic behaviour of electrically actuated microbeams; they employed the shooting method along with the Galerkin technique in order to solve the equation of motion. Rokni et al. [20] employed the Fredholm integral equation in order to present an analytical closed-form solution for the size-dependent static pull-in behaviour of clamped-free and clamped–clamped electrostatic microactuators. Abdel-Rahman and Nayfeh [21] examined the dynamic response of a resonant sensor under superharmonic and subharmonic electric actuations employing the method of multiple scales. Chaterjee and Pohit [22] developed a reduced-order model to examine the static and dynamic behaviours of electrically actuated microcantilevers separated by relatively large gaps. Moghimi Zandi and Ahamdian [23] examined the response of electrostatically actuated microstructures taking into account nonlinear squeeze film damping and in-plane forces. Jia et al. [24] examined the nonlinear resonant dynamics of electrically actuated microswitches, employing the method of averaging based on a single-mode Galerkin approximation. Kim et al. [25] contributed to the field by examining the nonlinear resonant behaviour of a microcantilever with a tip mass under an electrostatic excitation. Belardinelli et al. [26] continued the investigations by examining the dynamical behaviour of an electrically actuated microbeam under the effects of squeeze-film and thermoelastic damping.

1.5. Contributions of the current study to the field

The current paper, for the first time, analyses the *nonlinear size-dependent* electro-elasto-mechanics of a MEMS with an *initially curved deformable electrode based on the modified couple stress theory* via an efficient numerical technique. The nonlinear deformations, pull-in instabilities, and snap-through motions [27–29] are analysed for the deformable electrode actuated by both DC and AC voltages. This is accomplished through use of Hamilton's principle together with the modified couple stress theory, yielding the continuous model of the system with mechanical, electrostatic, as well as electro-dynamic nonlinearities. The Galerkin scheme is employed to obtain a *high-dimensional* reduced-order model of the system. Considering a high-dimensional reduced-order model is usually neglected in most studies in the literature, since it is numerically expensive and the run-time grows substantially with the number of degrees of freedom; however, not including higher dimensions yields inaccurate results and non-convergent computer codes for higher deflection amplitudes–as we shall see, employing a high-dimensional model gives the possibility of obtaining the response amplitude when the system changes behaviour from a hardening to a softening type. This reduced-order model is solved numerically, for the system subject to both DC and AC voltages, through use of the pseudo-arclength continuation method. The electro-elasto-static deformation amplitude versus the DC voltage and the pull-in voltages are obtained (in the absence of the AC voltage). Moreover, the electro-elasto-dynamics and snap-through motions are obtained when the system is subject to simultaneous DC and AC voltages; for larger AC voltages, the transition behaviour form an initial hardening to a secondary softening behaviour is discussed. A stability analysis is conducted via the Floquet theory. Finally, the importance of taking into account the modified couple stress theory, capable of capturing small-size effects, on the electro-elasto-static and electro-elasto-dynamic responses of the system is highlighted.

2. Continuous model based on modified couple stress theory

Shown in Fig. 1 is a MEMS resonator with a clamped–clamped initially curved deformable electrode. This electrode is modelled as an initially curved electrically actuated Euler–Bernoulli microbeam of length L, width b, thickness h, flexural stiffness EI, and axial stiffness EA; d represents the gap width and $u(x,t)$ and $w(x,t)$ denote the longitudinal and transverse displacements, respectively. The initial slight curvature in the transverse direction is denoted by $w_0(x)$; an electric actuation, comprised of DC and AC voltages, is applied to the deformable electrode in the form of $V_{DC} + V_{AC}\cos(\omega t)$, where ω is the frequency of the AC voltage. In the present study, it is assumed that $L \gg h$ and $L \gg w_0$, so that Euler–Bernoulli beam model can be used.