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Implicit implementation and consistent tangent modulus of a viscoplastic model for polymers

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ABSTRACT

In this work, the phenomenological viscoplastic DSGZ model (Duan et al., 2001 [13]), developed for glassy or semi-crystalline polymers, is numerically implemented in a three-dimensional framework, following an implicit formulation. The computational methodology is based on the radial return mapping algorithm. This implicit formulation leads to the definition of the consistent tangent modulus which permits the implementation in incremental micromechanical scale transition analysis. The extended model is validated by simulating the polypropylene thermoplastic behavior at various strain rates (from $0.92\ s^{-1}$ to $258\ s^{-1}$) and temperatures (from 20 °C to 60 °C). The model parameters for the studied material are identified using a heuristic optimization strategy based on genetic algorithm. The capabilities of the new implementation framework are illustrated by performing finite element simulations for multiaxial loading.

1. Introduction

Semi crystalline polymers are well known to exhibit a rate and temperature dependent behavior. With the increase interest for this kind of materials, in particular in the automotive industry, many phenomenological models have been developed [29,5,10,21,12,16,23,3,36,4,1] in order to take into account these properties. Many studies have also been performed to identify the evolution of damage in polymers and polymeric composites [25,26,37,2].

Several researches have proposed models to account for the viscoplastic behavior of polymers and they have developed appropriate implementation techniques for numerical calculations [32,28,31]. Some of the modeling efforts are focusing on semi-crystalline [39], glassy [35] or amorphous polymers [15].

Among the modeling efforts, the DSGZ model developed initially by [13] shows very interesting features and capabilities for viscoplasticity of polymers. Indeed, the DSGZ formulation is based on four previous models and it is able to trace different types of polymer behavior as the yielding and the hardening or softening of polymers.

Its initial one-dimensional form has been extended in 3 dimensions and implemented numerically following an explicit formulation [14]. The purpose of this paper is to propose for the first time a new, numerically implicit, formulation of the three-dimensional DSGZ phenomenological viscoplastic model and to implement it in the finite element software ABAQUS. Such an implementation allows us to use the DSGZ model as a constitutive model for matrix material in an incremental micromechanical analysis of glass fiber reinforced thermoplastic composites. Indeed, such homogenization schemes require the expression of the tangent modulus. This requirement is fulfilled using an implicit numerical integration scheme to compute the consistent tangent modulus at every step of the analysis by integrating the strain rate and the temperature effect on the matrix.

To perform numerical studies, appropriate DSGZ model parameters are identified experimentally on a thermoplastic material, namely polypropylene (PP), at different strain rates and temperatures. Certain methodologies have been proposed in the literature to identify viscoelastic/viscoplastic material parameters for polymers [22,40]. In this work, the parameter identification is achieved using a genetic algorithm coupled to gradient-based methods, which was applied successfully for shape memory alloys [34,8]. The experimental identification and validation of the model are based on thermomechanical tensile tests. Then its capability to simulate multiaxial loading is demonstrated.

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This paper is structured as follows: the first part is dedicated to a brief reminder of the background of this interesting model. The second part presents the numerical implicit formulation and the computation of the consistent tangent modulus, allowing the formulation of an algorithm for the finite element code ABAQUS. The next part focuses on two aspects: the identification of the model parameters for polypropylene material (PP) and the experimental validation by comparison with stress–strain curves obtained at different strain rates and temperatures. The fourth part of this paper is devoted to the application of the model by simulating multiaxial tensile-shear loading cases. These simulations are performed for 6 strain rates and 3 temperatures. Finally, the last part is dedicated to the application of the model on a dynamic load simulation. The aim of this part is to illustrate the capability of the implemented implicit model to be utilized for structural FE analysis.

2. DSGZ model background

The DSGZ is a viscoplastic phenomenological model developed for glassy or semi-crystalline polymers. It has the advantage to take into account the effect of the strain ε, the strain rate $\dot{\varepsilon}$, the temperature T, the softening and the hardening. According to the initial DSGZ constitutive law, the stress, σ, is given by

$$\sigma(\varepsilon,\dot{\varepsilon},T)=K[\mathbb{f}(\varepsilon)+[\mathbb{q}(\varepsilon,\dot{\varepsilon},T)-\mathbb{f}(\varepsilon)]\mathbb{r}(\varepsilon,\dot{\varepsilon},T)]\mathbb{h}(\dot{\varepsilon},T), \tag{1}$$

with

$$\mathbb{f}(\varepsilon)=\left[e^{-C_1\varepsilon}+\varepsilon^{C_2}\right]\left[1-e^{-\alpha\varepsilon}\right],\quad \mathbb{h}(\dot{\varepsilon},T)=\dot{\varepsilon}^m e^{a/T},$$

$$\mathbb{q}(\varepsilon,\dot{\varepsilon},T)=\frac{\varepsilon e^{[1-(\varepsilon/C_3\mathbb{h}(\dot{\varepsilon},T))]}}{C_3\mathbb{h}(\dot{\varepsilon},T)},\quad \mathbb{r}(\varepsilon,\dot{\varepsilon},T)=e^{[\ln(\mathbb{h}(\dot{\varepsilon},T))-C_4]\varepsilon}, \tag{2}$$

where K, C_1, C_2, C_3, C_4, a, α and m are the model constants.

Eq. (1) is based on four previously developed models, namely the Johnson–Cook, the G'Sell–Jonas, the Matsuoka, and the Brooks models. The model proposed by [6] is a constitutive law for dynamically recrystallizable materials. DSGZ model adopts a similar structure to Brooks model but the functions $\mathbb{f}$, $\mathbb{q}$, $\mathbb{h}$ and $\mathbb{r}$ are different. G'Sell and Jonas [17] developed a phenomenological model for semi-crystalline polymers, which has the advantage of integrating the effects of viscoelasticity and viscoplasticity in a single equation. This aspect is taken into account in the DSGZ model through the term $\mathbb{h}(\dot{\varepsilon},T)$. Johnson and Cook [27] proposed a simple model to describe the plastic behavior of ductile materials. Such behavior is integrated in Eq. (1) using the term $\mathbb{f}$. Finally, Matsuoka model [7] describes the behavior of glassy polymers. It includes the effects of nonlinear viscoelasticity, elasticity and the softening, but it does not account properly large deformations mechanisms. The authors of the DSGZ model used a simplified form of Matsuoka model to describe the behavior jump exhibited at the yield point of glassy polymers.

It is worth mentioning that the main purpose of the present paper is to provide a numerical formulation of a proper viscoplastic model for polymers and the inherent tangent modulus computation. Hence, the DSGZ model is chosen here as an illustrative implementation example. Further details and insights about the mathematical formulation of the model (in particular the strain rate, strain and temperature sensitivities of functions $\mathbb{f}$, $\mathbb{q}$, $\mathbb{h}$ and $\mathbb{r}$) and material parameters K, C_1 to C_4, a and α can be found in [13,14].

3. 3D extension of the constitutive model

The one-dimensional version of the DSGZ model has been extended to 3D by the same authors [14]. In this section the essential points of the three-dimensional version are discussed.

In elasto-plasticity and elasto-viscoplasticity, it is customary to separate the strain tensor, $\boldsymbol{\varepsilon}$, into an elastic, $\boldsymbol{\varepsilon}^e$, and a plastic, $\boldsymbol{\varepsilon}^p$, contribution and also to connect the stress tensor $\boldsymbol{\sigma}$ and the elastic strain through the Hooke's law. In many cases, the nonlinear nature of these materials motivates us to write these kinds of relations in incremental or rate form [9], i.e.

$$\dot{\boldsymbol{\varepsilon}}=\dot{\boldsymbol{\varepsilon}}^e+\dot{\boldsymbol{\varepsilon}}^p, \tag{3}$$

$$\dot{\boldsymbol{\sigma}}=\boldsymbol{\mathcal{C}}:[\dot{\boldsymbol{\varepsilon}}-\dot{\boldsymbol{\varepsilon}}^p], \tag{4}$$

where $\boldsymbol{\mathcal{C}}$ denotes the fourth order elastic stiffness tensor. This formalism has two significant advantages:

1. It allows easier numerical implementation, since any computational scheme in elasto-plasticity and elasto-viscoplasticity requires iterative solution (for instance, a return mapping algorithm based on an elastic trial stress) and incremental application of the applied loading.
2. The rate form is applicable not only in small deformation processes but also in large strain problems. Many experimental results in elasto-plastic materials are expressed in true (Cauchy) stress versus true (logarithmic) strain. The expressions (3) and (4) are very common in the case of hypoelastic materials, where the $\dot{\boldsymbol{\sigma}}$ denotes an objective stress rate and $\dot{\boldsymbol{\varepsilon}}$ is the rate of deformation [30]. Thus, the formulation (3) and (4) can be used for the DSGZ model [13], which has been developed considering large deformation processes.

When considering isotropic behavior for the elastic part, Eq. (4) can be expressed as

$$\dot{\boldsymbol{\sigma}}=2\mu[\dot{\boldsymbol{\varepsilon}}-\dot{\boldsymbol{\varepsilon}}^p]+\left[\kappa-\tfrac{2}{3}\mu\right]\mathrm{tr}\dot{\boldsymbol{\varepsilon}}\boldsymbol{I}, \tag{5}$$

where tr{•} denotes the trace of a second order tensor, $\boldsymbol{I}$ is the second order identity tensor, μ is the shear modulus and κ is the bulk modulus. Alternatively, using Eq. (5), the deviatoric parts of the stress and the strain

$$\boldsymbol{s}=\boldsymbol{\sigma}-\tfrac{1}{3}\mathrm{tr}\boldsymbol{\sigma}\boldsymbol{I},\quad \boldsymbol{e}=\boldsymbol{\varepsilon}-\tfrac{1}{3}\mathrm{tr}\boldsymbol{\varepsilon}\boldsymbol{I}, \tag{6}$$

are connected, in a rate form, using the following relation:

$$\dot{\boldsymbol{s}}=2\mu[\dot{\boldsymbol{e}}-\dot{\boldsymbol{\varepsilon}}^p]. \tag{7}$$

The rate of plastic strains is defined by a relation of the form

$$\dot{\boldsymbol{\varepsilon}}^p=\dot{p}\boldsymbol{\Lambda}^p, \tag{8}$$

where $\dot{p}=\sqrt{2/3[\dot{\boldsymbol{\varepsilon}}^p:\dot{\boldsymbol{\varepsilon}}^p]}$ and $\boldsymbol{\Lambda}^p$ defines the direction of the plastic flow. In classical J_2 viscoplasticity, the direction tensor is given by

$$\boldsymbol{\Lambda}^p=\frac{3}{2}\frac{\boldsymbol{s}}{\overline{\sigma}}. \tag{9}$$

The scalar quantity $\overline{\sigma}$ denotes the *Mises equivalent stress*, given as the second invariant of $\boldsymbol{s}$ per $\overline{\sigma}=\sqrt{3/2[\boldsymbol{s}:\boldsymbol{s}]}$. The DSGZ model assumes for the yield criterion[1]

$$\Phi^p(\boldsymbol{\sigma},p,\dot{p},T)=\overline{\sigma}-\sigma_y(p,\dot{p},T)\le 0, \tag{10}$$

where σ_y is provided by (11) by substituting the strain and strain rate with p and $\dot{p}$ correspondingly

$$\sigma_y(p,\dot{p},T)=K[\mathbb{f}(p)+[\mathbb{q}(p,\dot{p},T)-\mathbb{f}(p)]\mathbb{r}(p,\dot{p},T)]\mathbb{h}(\dot{p},T), \tag{11}$$

[1] In the extended version of the model [14], the authors included the hydrostatic pressure in the yield criterion. In such case a generally formulated requirement $\Phi^p(\boldsymbol{\sigma},p,\dot{p},T)\le 0$ is needed.