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# Study of the effects of cubic nonlinear damping on vibration isolations using Harmonic Balance Method

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ABSTRACT

In the present study, Harmonic Balance Method (HBM) is applied to investigate the performance of passive vibration isolators with cubic nonlinear damping. The results reveal that introducing either cubic nonlinear damping or linear damping could significantly reduce both the displacement transmissibility and the force transmissibility of the isolators over the resonance region. However, at the non-resonance region where frequency is lower than the resonant frequency, both the linear damping and the cubic nonlinear damping have almost no effect on the isolators. At the non-resonance region with higher frequency, increasing the linear damping has almost no effects on the displacement transmissibility but could raise the force transmissibility. In addition, the influence of the cubic nonlinear damping on the isolators is dependent on the type of the disturbing force. If the strength of the disturbing force is constant and independent of the excitation frequency, then the effect of cubic nonlinear damping on both the force and displacement transmissibility would be negligible. But, when the strength of the disturbing force is dependent of the excitation frequency, increasing the cubic nonlinear damping could slightly reduce the relative displacement transmissibility and increase the absolute displacement transmissibility but could significantly increase the force transmissibility. These conclusions are of significant importance in the analysis and design of nonlinear passive vibration isolators.



## 1. Introduction

Vibration isolator is a device having suitable characteristics and is often inserted between the vibrating source and the system requiring protection to reduce the level of transmitted vibration. The design of isolators always presents a challenge because there are various criteria and indices, which design engineers have to take into account, and these indices are usually related to one another. The choice of a particular index generally depends on the type of excitations and applications. The assumption of linearity has been widely applied for the study of the performance characteristics of vibration isolators. Under the assumption of linearity, the design criteria and indices can explicitly be related to the design parameters [1–3], and this can considerably facilitate a design process. For example, Soliman and Ismailzadeh [2] analytically linked the transmissibility of linear isolators to the optimal values of mass, stiffness and damping ratios, and consequently related the system resonant characteristics to these parameters. An excellent review about early studies of linear isolator systems has been provided by Snowdon [3].

Recently, researchers have shown more and more interests in nonlinear isolators. This is because all isolators in shock and vibration systems are inherently nonlinear [4–6], and the existence of nonlinearities has to be taken into account in designs if a better performance is to be achieved. For example, Mallik et al. [4] experimentally verified that the restoring and damping forces of elastomeric isolators have to be described using a nonlinear model. Richards and Singh [5] found that rubber isolators have both nonlinear damping and nonlinear stiffness. As a result of this, nonlinear isolators have been extensively studied by using both analytical and numerical approaches [7–10]. Chandra et al. [7] studied the transient responses of nonlinear, dissipative shock isolators using perturbation method and Laplace transform. They also tried to improve the performance of a nonlinear shock isolator by comparing the behaviors of four different alternatives [8]. Popov and Sankar [9] studied the effect of nonlinear orifice type damping on the response of a vibration isolator and found the nonlinear damping can cause a significant shift of the resonant frequency to a smaller value. Ravindra and Mallik [10]

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parametrically investigated the effects of various types of damping on the performance of nonlinear vibration isolators under harmonic excitations. In addition, different methods have been proposed to optimize the designs of nonlinear isolators [11–14]. Nayfeh et al. [11] proposed an optimal design method based on the concept of localized nonlinear normal modes. Royston and Singh [12] proposed an analytical framework for the optimization design of mounting systems where nonlinear effects were taken into account. Deshpande et al. [13] investigated the jump avoidance condition for the secondary suspension of a piecewise linear vibration isolation system, and used the RMS method to optimize the secondary suspension within a no jump zone. Jazar et al. [14] studied the jump avoidance condition for the design of a nonlinear passive engine mount. A more comprehensive review can be found in [15].

For the studies of nonlinear vibration isolators, lumped-parameter mathematical models incorporating $n$th power stiffness and damping characteristic are usually used [4,5,7–10,16]. Especially, the vibration isolators with cubic nonlinearity [7–10,17,18] have drawn particular attentions. More recently, using the concept of the Output Frequency Response Functions (OFRFs) [19], the authors [20] have revealed that, for vibration isolators, a cubic nonlinear damping characteristic can produce an ideal vibration isolation in terms of force transmissibility, such that only the force transmissibility over the resonant region is modified by the cubic damping but is almost unaffected over the non-resonant regions of frequencies. In this study, from a different perspective, a comprehensive study is carried out by using Harmonic Balance Method [21–23] to investigate the effects of linear damping and nonlinear cubic damping on the force and displacement transmissibilities of vibration isolators. Two scenarios are considered: the strength of disturbing force is constant and independent of the excitation frequency, and the strength of disturbing force is proportional to the square of exciting frequency. Important conclusions have been obtained, and they are of significant importance in the analysis and design of nonlinear passive vibration isolators, and can be used as a guideline for the design and selection of mounts or isolators in engineering applications.

## 2. Vibration isolators with a cubic nonlinear damping

Lumped-parameter mathematical models are frequently used in the analysis and design of vibration isolation systems as well as in the interpretation of characteristics of vibrating mechanical systems. Schematic diagrams of such idealized passive vibration isolation systems are illustrated in Fig. 1.

The essential features of the passive vibration isolators are a resilient load supporting mechanism (*stiffness*) and an energy-dissipating mechanism (*damping*). The stiffness and damping functions could be provided by a single element or by separate elements. When separate elements are employed, relative

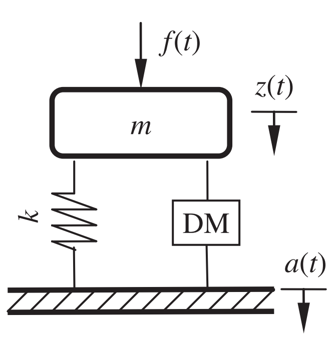


**Fig. 1.** Lumped-parameter representation of idealized vibration isolator.

undamped springs are often used with auxiliary damping elements. In this study, as shown in Fig. 1, the vibration isolator consists of a parallel combination of a linear spring whose stiffness is $k$ and a damping mechanism DM, and the equation of motion for the isolation system can be written as

$$m\ddot{z}(t)+F_{DM}+k\delta(t)=f(t) \quad (1)$$

where $z(t)$ is the absolute displacement of the isolated mass $m$, $\delta(t)=z(t)-a(t)$ is the relative displacement across the isolator, and vibration excitation is represented by the force $f(t)=A_0\cos(\omega t)$ or the foundation displacement $a(t)=A_0\cos(\omega t)$ or its derivatives. As the stiffness of the spring $k$ is assumed to be independent of displacement, the elastic restoring force $k\delta(t)$ is linearly proportional to the relative displacement. However, the damping mechanism is assumed to be with a nonlinear cubic damping curve, and the restoring damping force $F_{DM}$ is given as

$$F_{DM}=c\dot{\delta}(t)+c_3\dot{\delta}^3(t) \quad (2)$$

where $c_3>0$ are the nonlinear damping characteristic parameters of the system.

When the vibration excitation is the foundation displacement $a(t)$, the primary responses of interest for vibration isolator are the absolute displacement of the mass $z(t)$ and the relative displacement $\delta(t)$. In this case, Eq. (1) can be rewritten as

$$m\ddot{\delta}(t)+c\dot{\delta}(t)+c_3\dot{\delta}^3(t)+k\delta(t)=-m\ddot{a}(t) \quad (3)$$

When the vibration excitation is the force $f(t)$, Eq. (1) can be rewritten as

$$m\ddot{z}(t)+c\dot{z}(t)+c_3\dot{z}^3(t)+kz(t)=f(t) \quad (4)$$

In this case, the primary responses of interest are the force transmitted to the foundation $F_T$, given by

$$F_T(t)=f(t)-m\,\ddot{z}(t)=c\dot{z}(t)+c_3\dot{z}^3(t)+kz(t) \quad (5)$$

By setting $x=\delta/A_0$, $y=z/A_0$, $\Gamma=F_T/A_0$, $\tau=\omega_0 t$, $\omega_0=\sqrt{k/m}$, $\lambda=\omega/\omega_0$, $\xi=c/\sqrt{km}$, $\xi_{(2p+1)}=c_{(2p+1)}A_0^{2p}/\sqrt{(km)^{2p+1}}$, then Eqs. (3)–(5) can be rewritten as Eqs. (6)–(8) respectively, as follows:

$$\ddot{x}(\tau)+\xi\dot{x}(\tau)+\xi_3\dot{x}^3(\tau)+x(\tau)=\lambda^2\cos(\lambda\tau) \quad (6)$$

$$\ddot{y}(\tau)+\xi\dot{y}(\tau)+\xi_3\dot{y}^3(\tau)+y(\tau)=\cos(\lambda\tau) \quad (7)$$

$$\Gamma(\tau)=\xi\dot{y}(\tau)+\xi_3\dot{y}^3(\tau)+y(\tau) \quad (8)$$

Eq. (6) shows that the foundation displacement excitation can be equivalent to a force excitation where the strength of disturbing force is proportional to the square of exciting frequency. In some other applications, the isolation systems can also be expressed as Eq. (6), for example the engine mounts for rotating machines where the imbalance force [24] is the main vibration excitation source and its strength is dependent on rotating speed. Eqs. (6) and (7) can be uniformly rewritten as follows,

$$\ddot{\Delta}(\tau)+\xi\dot{\Delta}(\tau)+\xi_3\dot{\Delta}^3(\tau)+\Delta(\tau)=A\cos(\lambda\tau) \quad (9)$$

where $A=\lambda^2$ for the foundation displacement case and $A=1$ for the force excitation case, and the normalized force transmitted to foundation then can be rewritten as

$$\Gamma(\tau)=\xi\dot{\Delta}(\tau)+\xi_3\dot{\Delta}^3(\tau)+\Delta(\tau)=A\cos(\lambda\tau)-\ddot{\Delta}(\tau) \quad (10)$$

Moreover, when absolute displacement is considered for the foundation displacement excitation case, the vibration transmissibility can be measured through

$$\bar{z}(\tau)=\Delta(\tau)+\cos(\lambda\tau) \quad (11)$$

A good displacement excitation isolator should possess of low displacement transmissibility, measured by $\Delta(\tau)$ or $\bar{z}(\tau)$. Similarly,