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International Journal of Pressure Vessels and Piping 84 (2007) 132–138

INTERNATIONAL JOURNAL OF Pressure Vessels and Piping

www.elsevier.com/locate/ijpvp

Magnetothermoelastic interactions in hollow structures of functionally graded material subjected to mechanical loads

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Received 26 August 2005; received in revised form 29 September 2006; accepted 3 October 2006

Abstract

This paper considers the magnetothermoelastic problem of functionally graded material (FGM) hollow structures subjected to mechanical loads. Exact solutions for stresses and perturbations of the magnetic field vector in FGM hollow cylinders and FGM hollow spheres are determined using the infinitesimal theory of magnetothermoelasticity. The material stiffness, thermal expansion coefficient and magnetic permeability are assumed to obey the same simple power-law variation through the structures' wall thickness. The aim of this research is to understand the effect of composition on magneto- thermoelastic stresses and to design optimum FGM hollow cylinders and hollow spheres.

Keywords: FGM; Magnetothermoelastic; Hollow structure; Perturbation of magnetic field vector

1. Introduction

Functionally graded materials (FGM) that decrease thermal stresses have been developed for structural components or mechanical elements such as those used in nuclear, aircraft and space engineering. The FGM is a type of nonhomogeneous material whose composition is changed continuously. There are many ways to change the composition. It is expected that these materials will be particularly useful in high-temperature applications. To aid in the design of these materials, it would be useful to have an understanding of the manner in which the property gradients affect the induced thermal stresses. To date there have only been a few studies of the magnetothermoelastic behaviour of functionally gradient materials.

Durodola and Adlington [1] presented the use of numerical methods to assess the effect of various forms of gradation of material properties to control deformation and stresses in rotating axisymmetric components such as disks and rotors. Nadeau and Ferrari [2] presented a one-dimensional thermal stress analysis of a transversely isotropic layer that was inhomogeneous in its thickness. Using the infinitesimal theory of elasticity, Naki and Murat [3] obtained closed-form solutions for stresses and displacements in functionally graded cylindrical and spherical vessels subjected to internal pressure. Fukui et al. [4] studied the problem of uniform heating of a radial inhomogeneous thick-walled cylinder. Kwon et al. [5] studied the case of a graded sphere under non-uniform temperature variations by using a numerical integration procedure. Obata and Noda [6] used a perturbation approach to study the thermal stresses in a functionally graded hollow sphere and cylinder that was uniformly heated. Sugano [7] presented an analytical solution for the thermal stresses in a hollow cylinder whose Young's modulus and thermal expansion coefficient varied with radius but whose Poisson's ratio was constant. Lutz and Zimmerman [8] solved the problem of the uniform heating of a spherical body whose elastic modulus and thermal expansion coefficients each vary linearly with radius. However, investigations of exact solutions for FGM hollow cylinders and spheres, placed in a uniform magnetic field and temperature field, subjected to mechanical loads have not been found in the literature.

The present paper, by employing simplifying assumptions, presents simple, tractable closed-form solutions for a FGM hollow cylinder and a FGM hollow sphere. Closed-

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0308-0161/$ - see front matter
doi:10.1016/j.ijpvp.2006.10.001

Nomenclature

$\vec{U}$, u	displacement vector and radial displacement (m)
c_{ij}	elastic constants (N/m^2)
α_r, α_θ	thermal expansion coefficients (1/K)
σ_r, σ_θ	components of stresses (N/m^2)
T	temperature change (K)
ρ, t	mass density (kg/m^3) and time variable (s)
r	radial variable (m)
$\vec{H}$	magnetic intensity vector
$\vec{h}$	perturbation of magnetic field vector
$\vec{J}$	electric current density vector
$\vec{e}$	perturbation of electric field vector
μ	magnetic permeability (H/m)
τ_z, τ_ϕ	electro-magnetic stress (N/m^2)
a, b	inner and outer radii of the FGM hollow cylinder and the FGM hollow sphere (m)

form solutions for the through thickness stresses and the perturbation of the magnetic field vector in the FGM hollow cylinder and FGM hollow sphere are obtained. The model leads to solution of the standard Euler–Cauchy equation. The aim of this research is to understand the effect of the volumetric ratio of constituents and porosity on magnetothermoelastic stresses and perturbation of magnetic field vector and to design optimum FGM hollow cylinders and FGM hollow spheres.

2. Basic formulations and solutions

2.1. FGM hollow cylinder

Consider a long, FGM hollow cylinder with perfect conductivity placed in a uniform magnetic field $\vec{H}(0,0,H_z)$, letting the cylindrical coordinates of any representative point be (r, θ, z), and assuming that the FGM hollow cylinder is subjected to a rapid change in temperature T and mechanical loads. For the axisymmetric plane strain problem, the constitutive relations are

$$\sigma_r = c_{11}\frac{\partial u}{\partial r} + c_{12}\frac{u}{r} - \lambda_1 T, \quad \sigma_\theta = c_{12}\frac{\partial u}{\partial r} + c_{11}\frac{u}{r} - \lambda_2 T, \tag{1a,b}$$

$$\lambda_1 = c_{11}\alpha_r + c_{12}\alpha_\theta, \quad \lambda_2 = c_{12}\alpha_r + c_{22}\alpha_\theta, \tag{1c}$$

where $c_{ij}(i,j=1,2)$ and $\alpha_i(i=r,\theta)$ are elastic constants and thermal expansion coefficients, respectively, $\sigma_i(i=r,\theta)$ are the components of stress.

Assuming that the magnetic permeability, μ, [9] of the FGM hollow cylinder equals the magnetic permeability of the medium around it, assuming the medium to be non-ferromagnetic and non-ferroelectric and ignoring the Thompson effect the simplified Maxwell equations [10,11] of electrodynamics for a perfectly conducting elastic medium are

$$\vec{J} = \nabla\times\vec{h}, \quad \nabla\times\vec{e} = -\mu(r)\frac{\partial\vec{h}}{\partial t}, \quad \text{div}\,\vec{h} = 0$$
$$\vec{e} = -\mu(r)\left(\frac{\partial\vec{U}}{\partial t}\times\vec{H}\right), \quad \vec{h} = \nabla\times(\vec{U}\times\vec{H}). \tag{2}$$

Applying an initial magnetic field vector $\vec{H}(0,0,H_z)$ in cylindrical coordinates (r, θ, z) to Eq. (2), yields

$$\vec{U} = (u,0,0), \quad \vec{e} = -\mu(r)\left(0, H_z\frac{\partial u}{\partial t}, 0\right), \tag{3a}$$

$$\vec{h} = (0,0,h_z), \quad \vec{J} = \left(0, -\frac{\partial h_z}{\partial r}, 0\right), \quad h_z = -H_z\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right). \tag{3b}$$

The equilibrium equation of the FGM hollow cylinder, in the absence of body forces, is expressed as

$$\frac{\partial\sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} + \frac{\partial\tau_z}{\partial r} = 0, \tag{4}$$

where Maxwell's electro-magnetic stress τ_z is given by [10,12]

$$\tau_z = \mu(r)(\vec{J}\times\vec{H}) = \mu(r)H_z^2\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right). \tag{5}$$

The boundary conditions are

$$\sigma_r|_{r=a} = p_a, \quad \sigma_r|_{r=b} = p_b, \tag{6}$$

where a and b are the inner and outer radii of the FGM hollow cylinder, respectively.

It is now considered that all material coefficients have the same power function along the radial direction, i.e.

$$c_{11} = \bar{c}_{11}r^\beta, \quad c_{12} = \bar{c}_{12}r^\beta, \quad \alpha_r = \bar{\alpha}_r r^\beta,$$
$$\alpha_\theta = \bar{\alpha}_\theta r^\beta, \quad \mu(r) = \bar{\mu}r^\beta. \tag{7}$$

Here, $\bar{c}_{11}$, $\bar{c}_{12}$, $\bar{\alpha}_r$, $\bar{\alpha}_\theta$ and $\bar{\mu}$ are constants, and β is a parameter representing the inhomogeneous variations. Similar assumptions can be found in previous studies [13–15]. The range $-2\leqslant\beta\leqslant 2$ to be used in the present study covers all the values of coordinate exponents encountered in the references cited earlier. However, these values for β do not necessarily represent a certain material. Various values of β are used to demonstrate the effect of inhomogeneity on the stresses and perturbation of magnetic field vector distributions.

By means of Eq. (7), substituting Eqs. (1) and (5) into Eq. (4), yields

$$r^2\frac{\partial^2 u}{\partial r^2} + (1+\beta)r\frac{\partial u}{\partial r} + (I\beta - 1)u = JrT, \tag{8}$$