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# Efficient analysis of large-scale structural problems with geometrical non-linearity

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ABSTRACT

The paper presents an efficient methodology for the analysis of large-scale structural problems with geometrical non-linearity. A finite element based tool is developed, taking advantage of the analytical formulation of the stiffness matrix of a beam element, which is explicitly separated in linear and non-linear terms. The methodology proposes the substitution of the typical Newton-type non-linear analysis procedure, by a series of incremental linear analyses and a set of 'fictitious' forces, replacing the non-linear effect. The proposed technique is demonstrated in several structural problems that exhibit geometrical non-linear behaviour, with satisfactory results. The method's advantages on the analysis of large-scale non-linear problems are discussed, as well as the limitations and the further development that is required.



## 1. Introduction

One of the biggest challenges for the engineers, especially in the aeronautics, marine and automotive sectors, is to design and produce a reliable and cost-effective structure in a fast and efficient way. The design, analysis and certification process of a new product involves a great number of structural tests, which are performed in parallel with respective structural analysis; the requirements in complexity and size vary from the simple coupon level to the full-scale level. It is worth mentioning, that if testing could be reduced, based on a validated and safe Virtual Testing (numerical analysis) process, a reduction of 30–40% of component tests and consequently certification costs and a reduction of 20–30% of structural certification duration and thus time-to-market of a new product could be realized [1,2].

In order for these expectations to be fulfilled, advances in numerical analysis methodologies should be developed; this would enable the precise prediction of structural behaviour of large-scale components with high degree of complexity by taking into account the local non-linear behaviour, which is crucial for damage initiation and progression. A successful numerical model of this kind would possibly render the replacement of some tests by simulation at the component scale a feasible action, leading to great saving in the cost of large scale components, e.g. a new aircraft. Similar conclusion may be drawn for other typical large-scale structures, such as boats, cars, civil engineering constructions, etc.

Nowadays, there are many different approaches to the problem of large scale numerical simulation, basically based on large-scale Finite Element (FE) models of high degree of complexity. However, presently, existing finite element software capabilities are too limited for satisfying the simultaneous demand for non-linear analysis with millions of Degrees of Freedom (DOFs). Despite the constant increase of computing capabilities, allowing users to solve larger-scale problems in less time, there is a parallel tendency, focusing in the efficient use of shared-memory multi-processors. The main idea behind this method is the distribution of the computing 'effort' to different parallel remote processors, by supervising the exchange of data, as well as the priority of procedures [2,3]. The above method is capable of solving non-linear problems with good results [4], but its level of effectiveness depends mainly on the number of available CPU resources.

Another generic approach to the analysis of large scale FE models is the submodelling technique, i.e. condensing the mesh of the model at the areas of interest, which are considered to be critical, either at the design phase or during the solution phase, e.g. by application of the adaptive meshing techniques [5], or mesh superposition approaches [6]. An interesting approach to the submodelling technique has been developed within the EU Project MUSCA [1], by proposing an alternative scheme for global–local solution via splitting the model into separate parts and incorporating different sub-meshes for crucial parts of a large structure. Furthermore, a series of researches have been conducted focusing in improving the efficiency of FE models [7], by

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refining the mesh of the whole model (h-version) [8–10] or by increasing the degree of the elements (p-version) [11–17], during the convergence detection procedure. Although in numerous studies the submodelling technique has been applied, currently it is not compatible with non-linear problems.

There are also a number of promising methods, grouped under the category substructuring, e.g. [18–20], including Finite Element Tearing and Interconnecting (FETI) [21,22] and Balanced Domain Decomposition (BDD) [21,22]. Taking advantage of the division of the model in areas and subsequently solving each of them, they aim in developing an 'intelligent' pattern for spending the computer resources. To overcome this limitation, an improved BDD methodology dealing with non-linear localization technique has been proposed within the European Project MUSCA framework [1], by Cresta et al. [23].

It is also worth mentioning, that most of the researchers' effort in coping with large scale numerical problems has been expended in improving the efficiency of the classical FE method, by two main approaches, depending on the part of the analysis under consideration. The first category seeks the most efficient formulation for the non-linear finite element equations and includes the total Lagrangian [24–30], the updated Lagrangian [28,29], the Eulerian (mainly used for the fluid mechanics analyses) and the co-rotational formulations [24,31–33]. The second category involves the search of the most efficient solution scheme of the non-linear problem [34] and namely includes the Newton–Rahpson method, the displacement control method [35–37], the work control method [38,39], the arc-length method [40–48] and the minimum residual displacement [45]. A comprehensive review of the above mentioned common numerical methods has been presented by Clarke and Hancock [46] and Chen and Lui [47]. Although excessive work has been carried out with great results on some occasions, the problem of solving non-linear problems in such a large scale persists.

In the present paper, with the aim to overcome the above difficulties and provide a partial solution to the problem of efficient large-scale non-linear analysis for the case of geometrical non-linearity, a finite element based tool for fast, efficient and user friendly analysis is developed. Considering the iterative-incremental linearised nature of all the classical solutions coping with non-linear analyses, a novel technique is proposed, consisting of incremental linear only analyses, in which fictitious forces are applied, in order to substitute the non-linear effect at each increment. This is accomplished by formulating the stiffness matrix in such a way, that the linear terms are explicitly separated from the non-linear terms, with the latter representing the provenance of the fictitious forces mentioned above. For the sake of simplicity, the stiffness matrix formulation is developed for a typical finite beam element, but with the proper adaptations may treat all type of elements. The main innovation deals with solving the non-linear problem with a sequence of linear solutions leading to a significant computational cost saving. The proposed methodology is demonstrated in the case of two-dimensional beam finite element models, but it can be extended to three-dimensions and to most of the other types of finite element analysis. Different non-linear problems are presented for demonstration purposes, in comparison with analytical, experimental and available finite element results from other authors. The limitations, as well as, the perspectives of the proposed methodology are discussed.

## 2. Derivation of stiffness matrix and load vector of a beam finite element with geometrical non-linearity

In general, structural non-linearity may arise either from geometrical non-linearity (e.g. large deflections, large strains, contact) or material non-linearity (e.g. elastic–plastic material behaviour, damage progression, etc.). The proposed methodology tackles the issue of geometrical non-linearity, but with proper adaptations may treat material non-linearity as well. Furthermore, the finite element implementation is demonstrated in the case of a beam as a basic element, taking advantage of its convenience in describing the geometrical non-linearity by analytical formulas, compared to more complicated shell or solid elements. The method is applied on typical problems of framed structures, in order to assess its efficiency.

### *2.1. Basic assumptions and governing equations of a beam subjected to non-linear bending*

A beam is a structural member with length to cross-sectional dimensions ratio higher than ten and it undergoes stretching along its length and bending about an axis transverse to the length, as shown in Fig. 1. A right-handed Cartesian coordinate system $(x, y, z)$ is chosen in such a way that $x$-axis coincides with the beam axis passing through the centroid of each cross-section. The coordinate axes $y$ and $z$ are the principal inertial axes of the cross-section.

The Euler–Bernoulli assumptions are adopted for the present formulation. For simplicity reasons, only the extensional-bending problem of a beam with large deflections and small strain in the two-dimensional domain $(x, z)$ is considered, while the torsional problem is not taken into account in the present formulations. Using the non-linear strain–displacement relations [49] and omitting the large strain terms of higher order than square, the axial strain is written as in Eq. (1), while all other strains are zero

$$\varepsilon_x = \frac{\partial u_0(x)}{\partial x} - z\frac{\partial^2 w_0(x)}{\partial x^2} + \frac{1}{2}\left(\frac{\partial w_0(x)}{\partial x}\right)^2 \tag{1}$$

The symbols $(u, v, w)$ denote the total displacements along the coordinate directions $(x, y, z)$, respectively, while $u_0$ and $w_0$ denote the axial and transverse displacements of a point on the beam neutral axis.

Provided that the final goal of the present derivation is to achieve a stiffness matrix, in which the linear and the non-linear components are explicitly separated, the total strain in Eq. (1) is dissociated in two distinct parts, namely the linear $\varepsilon_{xL}$ and the non-linear part $\varepsilon_{xNL}$, as follows:

$$\varepsilon_x = \varepsilon_{xL} + \varepsilon_{xNL}$$

$$\varepsilon_{xL} = \frac{\partial u_0(x)}{\partial x} - z\frac{\partial^2 w_0(x)}{\partial x^2}, \quad \varepsilon_{xNL} = \frac{1}{2}\left(\frac{\partial w_0(x)}{\partial x}\right)^2 \tag{2}$$

Before proceeding with the non-linear beam implementation, a typical beam finite element is considered, as shown in Fig. 2. The above mentioned element has two nodes and three Degrees of Freedom (DOFs), namely $u$, $w$, and $\theta$ at each node.

Following the finite element approximation, a displacement assumption within the element is performed, which gives the displacement at any point as a combination of the displacements

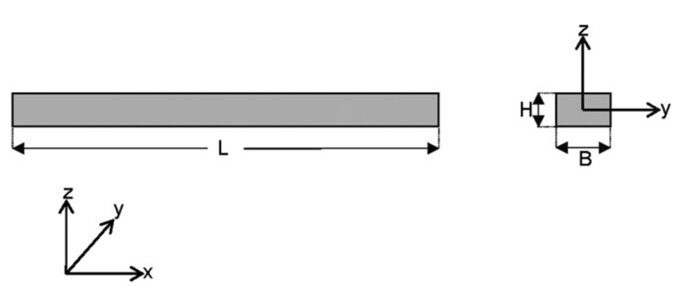


**Fig. 1.** Schematic of a typical beam configuration.