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The force transmissibility of MDOF structures with a non-linear viscous damping device

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ABSTRACT

In the present study, the concept of the Output Frequency Response Function (OFRF), recently proposed by the authors, is applied to theoretically investigate the force transmissibility of MDOF structures with a cubic non-linear viscous damping device. The results analytically show that the introduction of cubic non-linear damping can significantly reduce the transmissibility over all resonance regions for a Multiple Degree of Freedom (MDOF) structure and at the same time leave the transmissibility over the isolation region virtually unaffected. The analysis also indicates that a strong linear damping may shift the system resonances and compromise the beneficial effects of cubic non-linear viscous damping on the force transmissibility of MDOF structures. This suggests that a less significant linear damping together with a strong cubic non-linear damping can be used in MDOF structures to achieve a desired vibration isolation performance. This research work has a significant implication for the design of viscously damped MDOF structures for a wide range of practical applications.

1. Introduction

Vibration isolation is an effective method of reducing the transmission of vibration energy so as to protect equipment or structures from vibration disturbances [1]. Generally speaking, vibration isolation systems fall into two categories: passive and active [2], and passive isolation systems can be either linear or non-linear. The design of isolation systems always presents a challenge to mechanical engineers because various criteria and indices have to be considered in practice. For linear isolation systems, which have been widely studied in the literature, the isolation criteria and indices can often be explicitly expressed in term of the design parameters, such as damping and stiffness coefficients. This greatly facilitates the design process so that an optimal design of linear isolation systems can be achieved relatively easily. For example, Soliman and Ismailzadeh [3] analytically derived the relationship between the transmissibility and the mass, stiffness, and damping ratios for linear isolators to explicitly relate the system resonant characteristics to these parameters. Most recently, various powerful optimization techniques such as the recursive quadratic programming (RQP) technique [4], the sequential quadratic programming (SQP) technique [5] and genetic algorithms (GA) [6] have been applied to design linear isolation systems to achieve a better isolation performance. However, the design of non-linear isolation systems is much more complicated and is still a difficult challenge. The difficulty is that the analysis of non-linear systems is much more complicated, *per se*, since a closed-form analytic solution to non-linear differential equations is possible only for a limited number of special classes of non-linear differential equations [7]. Usually, researchers have to simplify the non-linear systems analysis by resorting to Single Degree of Freedom (SDOF) or low dimensional models. However, even with simplified models, the analysis of non-linear systems is still not an easy task. For the study of non-linear vibration isolation systems, an immediate difficulty is that it is hard to derive an explicit analytical description for the relationship between the system non-linear characteristic parameters and the transmissibility. As a result, most research effort has been focused on the analysis of relatively simple SDOF and 2-DOF non-linear isolators. In addition, for some non-linear vibration systems, the difficulties in analysis and design are also due to individualistic behaviors such as dependence on initial condition or energy and possible existence of multiple solutions and bifurcation phenomena. A very comprehensive survey about the recent developments of non-linear vibration isolators has been contributed by Ibrahim [8], in which many cited studies [9–16] have revealed that the introduction of non-linear damping and stiffness are really of great benefit in vibration isolation.

Although the design of linear isolation systems is relatively easier than the design of non-linear isolation systems, there is a well-known *dilemma* associated with viscously damped linear

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isolation systems. That is, when the linear viscous damping level is increased to reduce the transmissibility in the resonant regions, the transmissibility is increased in the isolation region where isolation is required. Most recently, by using the concept of Output Frequency Response Function (OFRF) [17,18], the authors [19] have analytically revealed that, for SDOF vibration isolators, a cubic non-linear viscous damping characteristic can produce an ideal vibration isolation such that only transmissibility over the resonant frequency region is reduced by the non-linear damping while the transmissibility over non-resonant frequency ranges is virtually unaffected. Therefore, by introducing cubic non-linear viscous damping to SDOF vibration isolators the dilemma or compromise associated with linear viscous damping isolators can be overcome. In the present study, these results are extended to investigate the force transmissibility of MDOF structures with a cubic non-linear damping device. The analysis theoretically proves that the introduction of a cubic non-linear viscous damping characteristic can significantly reduce the transmissibility around all resonant frequencies of MDOF structures but have virtually no effect on the transmissibility over the non-resonant frequency regions. Numerical simulation studies are carried out to verify the theoretical analysis and demonstrate the considerable engineering significance of the conclusions reached in this study. The revelation that a MDOF isolator with a cubic non-linear viscous damping characteristic possesses ideal vibration isolation properties provides an important foundation for the development of novel passive or semi-active solutions to vibration isolation problems of MDOF structural systems.

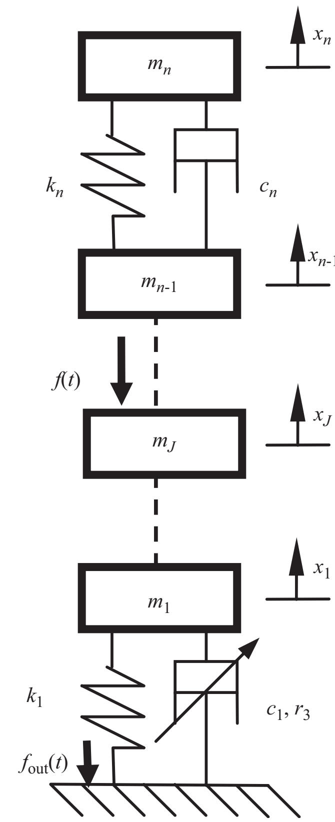

Fig. 1. MDOF structure on a support base with a cubic non-linear damping.

2. MDOF structures with a non-linear viscous damping characteristic

2.1. Governing motion equations

Consider a MDOF structure with a cubic non-linear viscous damping characteristic located at the degree of freedom at the bottom of the structure as shown in Fig. 1, where

$$f(t) = A\sin(\Omega t) \tag{1}$$

is the harmonic force acting on the Jth mass with frequency Ω and magnitude A, $f_{out}(t)$ is the force transmitted to the supporting base, and $x_i(t)$ is the displacement of mass i ($i=1, \ldots, n$). The damping force associated with the degree of freedom at the bottom of the structure is described by

$$f_{NL} = c_1\dot{x}_1 + r_3\dot{x}_1^3 \tag{2}$$

where r_3 is the cubic non-linear damping characteristic parameter. The governing equations of the MDOF structure can be written in the following matrix form

$$M\ddot{\mathbf{x}} + C\dot{\mathbf{x}} + \mathrm{K}\mathrm{x} + \mathbf{F}_N = \mathbf{F}(t) \tag{3}$$

where

$$\mathbf{M} = \begin{pmatrix} m_1 & 0 & \cdots & 0 \\ 0 & m_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & m_n \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} c_1+c_2 & -c_2 & 0 & \cdots & 0 \\ -c_2 & c_2+c_3 & -c_3 & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & -c_{n-1} & c_{n-1}+c_n & -c_n \\ 0 & \cdots & 0 & -c_n & c_n \end{pmatrix}$$

$$\mathbf{K} = \begin{pmatrix} k_1+k_2 & -k_2 & 0 & \cdots & 0 \\ -k_2 & k_2+k_3 & -k_3 & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & -k_{n-1} & k_{n-1}+k_n & -k_n \\ 0 & \cdots & 0 & -k_n & k_n \end{pmatrix}$$

are the system mass, damping and stiffness matrix, respectively, and

$$\mathbf{x} = (x_1, \ldots, x_n)', \quad \mathbf{F}(t) = (\overbrace{0,\ldots,0}^{J-1}, f(t), 0, \ldots, 0)', \text{ and}$$
$$\mathbf{F}_N = (r_3\dot{x}_1^3\ 0, \ldots, 0)'$$

In this study, the damping matrix $\mathbf{C}$ is assumed to be proportional to the stiffness matrix $\mathbf{K}$, e.g., $\mathbf{C}=\mu\mathbf{K}$ where μ is the damping ratio. The force transmitted to the supporting base $f_{out}(t)$ can be evaluated as follows,

$$f_{out}(t) = k_1x_1 + c_1\dot{x}_1 + r_3\dot{x}_1^3 \tag{4}$$

Denote $\mathbf{y}=(y_1, \ldots, y_n)'$ and $\mathbf{x}=\Phi\mathbf{y}$ where

$$\Phi = \begin{pmatrix} \Phi_{11} & \ldots & \Phi_{1n} \\ \vdots & \ddots & \vdots \\ \Phi_{n1} & \ldots & \Phi_{nn} \end{pmatrix} \tag{5}$$

is the mode shape matrix [20], which is generated by solving the following eigenvalue problem

$$(\mathbf{K} - \varpi^2\mathbf{M})\Phi = 0 \tag{6}$$

where ϖ denotes the eigenvalue of the system.

Multiplying Eq. (3) by Φ^T and then replacing $\mathbf{x}$ with $\Phi\mathbf{y}$ yields

$$\Phi^T\mathbf{M}\Phi\ddot{\mathbf{y}} + \Phi^T\mathbf{C}\Phi\dot{\mathbf{y}} + \Phi^T\mathbf{K}\Phi\mathbf{y} + \Phi^T\mathbf{F}_N = \Phi^T\mathbf{F}(t) \tag{7}$$