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Contact analysis of a semi-infinite strip pressed onto a half plane by a line force

T. Chaise [a,*], R.J.H. Paynter [b], D.A. Hills [b]

[a] Université de Lyon, CNRS, INSA-Lyon, LaMCoS UMR5259, F-69621, France
[b] Department of Engineering Science, The University of Oxford, Parks Road, Oxford OX1 3PJ, United Kingdom

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ABSTRACT

Receding contacts are a class of contacts that have received little attention. The first studies carried out on the subject have shown that the area of contact decreases as a load is applied and is load independent. The simple plane problem of an elastic layer pressed onto an elastically similar half space by a line force is studied here. The problem is solved for several coefficients of friction using distributions of edge dislocations. The extent of the stick and the slip zones are found and the lift-off angle of the strip is estimated from the resulting tractions distribution.

1. Introduction

Receding contacts constitute an important class of contact problems whose properties are only partly understood. Although the first studies were carried out by Keer et al. [6], Tsai et al. [9] and Dundurs [3] 40 years ago, they have still received comparatively little attention even though most 'high strength friction grip' connections (to use the nomenclature employed in civil engineering) are of this type. El-Borgi et al. [4] studied the case of frictionless contact between a layer and a homogeneous substrate while Ahn and Barber [1] recently studied a similar contact problem under cyclic loading. In particular we would like to understand the frictional damping properties, final contact size and contact pressure distribution.

A simple plane problem is an elastic layer resting on an elastically similar half-plane, and where an applied contact pressure is exerted which stops short of the ends of the strip. In the extreme case, we may think of the contact pressure being reduced to a 'knife edge', Fig. 1, and the layer as being infinitely long. This geometry is very attractive as a candidate for study, because the only length dimension in the problem, as shown, is the thickness of the layer, c, so that, together with the interfacial coefficient of friction, f, these constitute the only independent variables in the problem, and therefore a comprehensive solution should be possible, revealing some basic properties. We start by assuming that the layer and the substrate are in intimate contact, that the coefficient of friction is sufficient to prevent all slip, and that the contact pressure along the interface is compressive everywhere, so that the substrate and the layer combined constitute a homogeneous elastic monolithic half-plane. Note that (for reasons which will become clear) we set up axes centred on the substrate/layer interface. The application of a normal force, P, at the point $(0,c)$ induces the following contact pressure $p(x,0)$ and shear tractions $q(x,0)$ [7], represented in Fig. 2:

$$p(x,0)=\frac{-2P}{\pi c}\frac{c^4}{(c^2+x^2)^2} \tag{1}$$

$$q(x,0)=\frac{2P}{\pi c}\frac{xc^3}{(c^2+x^2)^2}. \tag{2}$$

We see that this 'bilateral' solution predicts closure everywhere and an extent of slip given by $f=x/c$, i.e. over the semi-infinite lines $fc\le|x|\le\infty$. Thus, an infinitely high coefficient of friction is required to ensure stick everywhere, and if this is achieved the contacting pair will not open. If the coefficient of friction is finite we expect slip over the intervals $a\le|x|\le\infty$, where a is to be found, and anticipating properties of the solution to be revealed that this will also cause opening over the interval $b\le|x|\le\infty$, where, again, b is to be found. The method we propose to adopt is to arrange distributions of edge dislocations along the interface line to restore conventional bilateral (Signorini) contact inequalities.

2. Formulation

Following Schmueser et al. [8] and Comninou et al. [2], we assume that the half-plane $y<c$ contains an array of dislocations.

* Corresponding author.
E-mail addresses: Thibaut.Chaise@insa-lyon.fr (T. Chaise), David.Hills@eng.ox.ac.uk (D.A. Hills).

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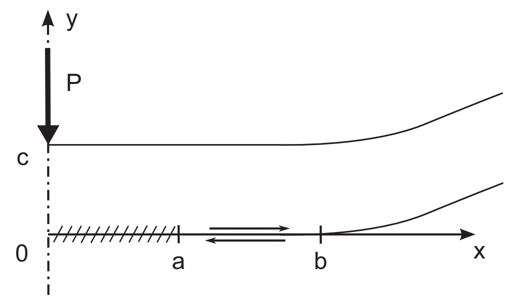

Fig. 1. Geometry of the problem.

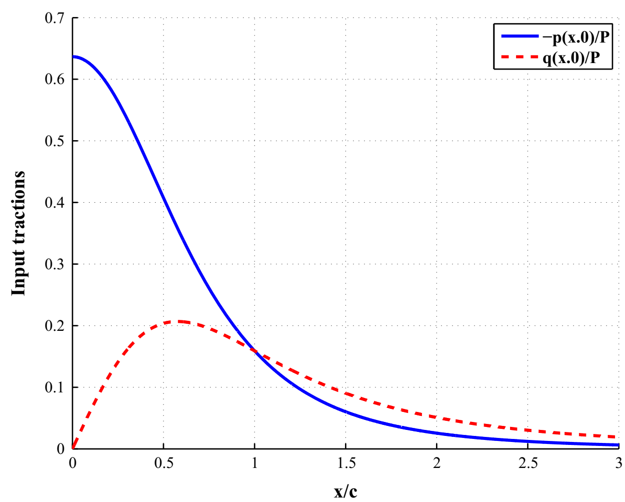

Fig. 2. Contact pressure and shear tractions at $(x,0)$ generated by a normal force P.

The state of stress induced by a dislocation with Burgers vector (b_x, b_y) located at point $(\xi, 0)$ is given by

$$\begin{Bmatrix} \sigma_{xy}(x,0) \\ \sigma_{yy}(x,0) \end{Bmatrix} = \frac{2\mu}{\pi(\kappa+1)} \begin{bmatrix} \frac{1}{x-\xi} + G_{xxy}(x,\xi) & G_{yxy}(x,\xi) \\ G_{xyy}(x,\xi) & \frac{1}{x-\xi} + G_{yyy}(x,\xi) \end{bmatrix} \begin{Bmatrix} b_x(\xi,0) \\ b_y(\xi,0) \end{Bmatrix} \tag{3}$$

where we have separated out the regular terms, $G_{ijk}(x,\xi)$, which are given in Appendix A, and account for the presence of the free surface, μ is the modulus of rigidity, and κ is Kolosov's constant. We know the tractions $p(x)$ and $q(x)$ arising along the line $y=0$ induced by the applied load, and can therefore write down the total tractions $N(x)$ and $S(x)$ (normal and shear, respectively) present along this line as

$$N(x) = p(x) + \frac{2\mu}{\pi(\kappa+1)} \left\{ \int_b^{\infty} \left[\frac{1}{x-\xi} + G_{yyy}(x,\xi) \right] B_y(\xi)\, d\xi + \int_{-\infty}^{-b} \left[\frac{1}{x-\xi} + G_{yyy}(x,\xi) \right] B_y(\xi)\, d\xi + \int_a^{\infty} G_{xyy}(x,\xi) B_x(\xi)\, d\xi + \int_{-\infty}^{-a} G_{xyy}(x,\xi) B_x(\xi)\, d\xi \right\} \tag{4}$$

$$S(x) = q(x) + \frac{2\mu}{\pi(\kappa+1)} \left\{ \int_a^{\infty} \left[\frac{1}{x-\xi} + G_{xxy}(x,\xi) \right] B_x(\xi)\, d\xi. + \int_{-\infty}^{-a} \left[\frac{1}{x-\xi} + G_{xxy}(x,\xi) \right] B_x(\xi)\, d\xi + \int_b^{\infty} G_{yxy}(x,\xi) B_y(\xi)\, d\xi + \int_{-\infty}^{-b} G_{yxy}(x,\xi) B_y(\xi)\, d\xi \right\}. \tag{5}$$

where $B_i(\xi) = db_i/d\xi, i = x, y$ are the dislocation densities. The first step in simplification of the integrals is to exploit the inherent symmetry and anti-symmetry of the influence functions for the dislocations, so that

$$b_x(\xi) = -b_x(\xi) \quad \text{or} \quad B_x(\xi) = B_x(\xi) \tag{6}$$

$$b_y(\xi) = b_y(\xi) \quad \text{or} \quad B_y(\xi) = -B_y(\xi) \tag{7}$$

and the corrective terms in the kernels having the following properties:

$$G_{yyy}(-x, -\xi) = -G_{yyy}(x, \xi) \tag{8}$$

$$G_{xyy}(-x, -\xi) = G_{xyy}(x, \xi) \tag{9}$$

$$G_{xxy}(-x, -\xi) = -G_{xxy}(x, \xi) \tag{10}$$

$$G_{yxy}(-x, -\xi) = G_{yxy}(x, \xi), \tag{11}$$

so that we may now write

$$N(x) = p(x) + \frac{2\mu}{\pi(\kappa+1)} \left\{ \int_b^{\infty} \left[\frac{1}{x-\xi} + F_{yyy}(x,\xi) \right] B_y(\xi)\, d\xi + \int_a^{\infty} F_{xyy}(x,\xi) B_x(\xi)\, d\xi \right\} \tag{12}$$

$$S(x) = q(x) + \frac{2\mu}{\pi(\kappa+1)} \left\{ \int_a^{\infty} \left[\frac{1}{x-\xi} + F_{xxy}(x,\xi) \right] B_x(\xi)\, d\xi + \int_b^{\infty} F_{yxy}(x,\xi) B_y(\xi)\, d\xi \right\} \tag{13}$$

where

$$F_{yyy}(x,\xi) \equiv -\frac{1}{x+\xi} + G_{yyy}(x,\xi) - G_{yyy}(x,-\xi) \tag{14}$$

$$F_{xyy}(x,\xi) \equiv G_{xyy}(x,\xi) + G_{xyy}(x,-\xi) \tag{15}$$

$$F_{xxy}(x,\xi) \equiv \frac{1}{x+\xi} + G_{xxy}(x,\xi) + G_{xxy}(x,-\xi) \tag{16}$$

$$F_{yxy}(x,\xi) \equiv G_{yxy}(x,\xi) - G_{yxy}(x,-\xi). \tag{17}$$

The next step is to write down the Signorini conditions for the contact, where we have chosen the appropriate sign for the shear traction to make it consistent with the underlying shear caused by the applied load:

$$N(x) = 0, \quad b \le x < \infty \tag{18}$$

$$S(x) = -\operatorname{sign}(x) f N(x), \quad a \le x < \infty \tag{19}$$

We note that the interval over which we impose the first of these conditions is precisely the same as the range of the singular integral for the climb dislocations ($B_y(\xi)$), so that

$$\frac{2\mu}{\pi(\kappa+1)} \left(\int_a^{\infty} F_{xyy}(x,\xi) B_x(\xi)\, d\xi + \int_b^{\infty} \left\{ \frac{1}{x-\xi} + F_{yyy}(x,\xi) \right\} B_y(\xi)\, d\xi \right) = -p(x), \quad b \le x < \infty \tag{20}$$

In Eq. (20) there is a solitary Cauchy integral, over the range $b \le x < \infty$, and this is the same as the interval over which it is imposed. We need to re-examine the second condition and re-cast it in the form

$$S(x) = -H(b - |x|)\operatorname{sign}(x) f N(x), \quad a \le x < \infty \tag{21}$$

to ensure that the range over which this is imposed and the interval of the singular integral are the same, giving

$$\frac{2\mu}{\pi(\kappa+1)} \left(\int_a^{\infty} \left[\frac{1}{x-\xi} + L_x(x,\xi) \right] B_x(\xi)\, d\xi + \int_b^{\infty} \left[L_y(x,\xi) + H(b-|x|)\operatorname{sign}(x) f \frac{1}{x-\xi} \right] B_y(\xi)\, d\xi \right)$$