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Vibration and aeroelastic properties of ordered and disordered two-span panels in supersonic airflow

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ABSTRACT

In this paper, the analysis of vibration and aeroelastic properties of ordered and disordered two-span panels is carried out. The equation of motion of each sub-panel of the two spans is formulated using Hamilton's principle. Supersonic piston theory is applied to evaluate the unsteady aerodynamic pressure. The partial differential equation of motion of the two-span panel is solved, and the mode shapes of the panels with and without aerodynamic pressure are obtained. The free vibration behaviors of the two-span panel are analyzed. Time-domain responses of the panel are computed by the mode superposition method using the mode shapes obtained previously. It is noted from the free vibration analysis that the vibration localization will happen on the disordered two-span panel. Aeroelastic analyses of ordered and disordered two-span panels are also carried out through the frequency-domain method. Characteristics of the aeroelastic stability and fluttering mode of the disordered two-span panel are analyzed. Simulation results show that the disorder of the two-span panel will decrease the critical flutter aerodynamic pressure of the structural system. The influences of the disorder degree on the vibration localization and the flutter bound of the two-span panel are investigated. The present results are useful for the analysis and design of the multi-span structures.

1. Introduction

Multi-span structures are usually used in the architectural and mechanical industries. For example, the long bridge and the surface of the aircraft can all be modeled as the multi-span beams or plates. The vibration behaviors of multi-span structures have drawn a lot of attentions of researchers. Veletsos and Newmark [1] were the first two researchers to investigate the vibration of plates with internal line supports. Xiang et al. [2] studied the vibration performances of multi-span rectangular plates applying the Levy method. Lv et al. [3] analyzed the exact solution of the free vibration of long-span continuous rectangular plates using the classical Kirchhoff plate theory. Li et al. [4] researched the wave localization in disordered periodic multi-span rib-stiffened plates applying the elastic dynamics theory. Li and Wang [5] studied the wave propagation and localization in randomly disordered periodic multi-span beams on elastic foundations. Cheung et al. [6] investigated the vibration of a multi-span non-uniform bridge subjected to a moving vehicle using the modified beam vibration mode functions as the assumed modes. Li et al. [7] researched the buckling mode localization in the periodic multi-span beam with disorder occurring in an arbitrary single span.

Xu and Huang [8] introduced a new random wave reflector in the transverse vibration and wave propagation control of an infinite multi-span simply supported beam. Yesilce and Demirdag [9] determined the exact solutions for the first five natural frequencies and mode shapes of a Timoshenko multi-span beam subjected to axial force. Golley and Petrolito [10] studied the dynamical properties of orthotropic plates with internal supports applying the finite strip element method. Kim and Dickinson [11] analyzed the free vibration characteristics of multi-span plates using a set of one-dimensional orthogonal polynomial functions. Dickinson and Warburton [12] calculated the natural frequencies of two-span plates using the edge effect method. Liew and Lam [13] computed the natural frequencies of multi-span plates using the orthogonally generated two-dimensional plate functions. Liew et al. [14] investigated the vibration behaviors of rectangular Mindlin plates with internal line supports either in parallel or in diagonal direction.

Flutter is one type of the self-excited oscillation. It is a subject which is mainly concentrated on the coupling effects of the aerodynamic load, elastic force and inertia force of the structure. Panel flutter as one kind of the flutter can be generated when a

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panel is exposed to airflow along its surface. A large amount of literatures have studied the characteristics of the panel flutter. Tubaldi et al. [15] researched the nonlinear vibration behaviors of thin infinitely long rectangular plates subjected to axial flow. The influences of the flow velocities were researched. Guo and Mei [16] studied the nonlinear flutter properties of panels with temperature change using the aeroelastic modes instead of the traditional natural modes in vacuo. Abbas et al. [17] investigated the effect of the system parameters on the flutter of a curved skin panel in supersonic and hypersonic flows. Kouchakzadeh et al. [18] conducted the panel flutter analysis of general laminated composite panels based on Galerkin's method. Zhou et al. [19] studied the flutter characteristics of nonlinear composite panels considering the thermal effect using the finite element method (FEM). Vedeneev [20] conducted a comprehensive numerical investigation of single mode flutter to determine the flutter boundaries and their transformations due to the parameter changes. Koo and Hwang [21] performed a study of flutter characteristics for composite panels with structural damping using the FEM. In recent years, we have studied the aeroelastic and aerothermoelastic characteristics of composite laminated panels and the active flutter and thermal buckling control of these structures [22–24].

Based on the above analysis, it is noted that although many literatures have studied the vibration behaviors of multi-span structures, and the panel flutter has been deeply investigated by a lot of researchers, few literatures have conducted the aeroelastic analysis of multi-span structures in supersonic airflow. Moreover, the vibrations and aeroelastic characteristics of disordered periodic multi-span structures are also lack of investigations. Inspired from these, vibrations and aeroelastic behaviors of ordered and disordered two-span panels are investigated in this study. The equation of motion of the two-span panel is formulated using Hamilton's principle. Supersonic piston theory is applied to evaluate the unsteady aerodynamic pressure. Mode shapes of the two-span panels with and without aerodynamic pressure are calculated. Time-domain responses of the panel are computed by the mode superposition method using the mode shapes obtained previously. Characteristics of the aeroelastic stability and fluttering mode of the disordered two-span panel are analyzed, and the influences of the disorder degree on the vibration localization and flutter bound are investigated. Some interesting and novel results are obtained.

2. Formulation for the equation of motion

Fig. 1 shows the schematic diagram of a two-span panel in the supersonic flow. The local coordinate of each span is also displayed in the figure. The lengths of the left and right spans in the x direction are a_1 and a_2, respectively. The width and thickness of each sub-panel are denoted by b and h. The free airflow is along the x direction. It is to be pointed that the positive direction of the x-axis is defined in the direction to the right for the left span, and it is defined in the direction to the left for the right span [25].

The two sub-panels have the same thickness which is relatively thin, so the Kirchhoff plate theory is applied in the structural modeling, and it can be expressed as

$$u=-z\frac{\partial w}{\partial x},\quad v=-z\frac{\partial w}{\partial y},\quad w=w, \tag{1}$$

where u, v and w are the in-plane and transverse displacements along the x, y and z directions, and z is the coordinate in the z-axis. The strain–displacement relation is given as

$$\boldsymbol{\varepsilon}=z\left[-\frac{\partial^2 w}{\partial x^2}\quad -\frac{\partial^2 w}{\partial y^2}\quad -2\frac{\partial^2 w}{\partial x\partial y}\right]^{\mathrm{T}}=z\boldsymbol{\kappa}, \tag{2}$$

where $\boldsymbol{\varepsilon}=[\varepsilon_x,\ \varepsilon_y,\ \varepsilon_{xy}]^{\mathrm{T}}$ is the strain vector, and $\boldsymbol{\kappa}$ is the bending curvature vector. The constitutive equation of each sub-panel is expressed as $\boldsymbol{\sigma}=[\sigma_x,\ \sigma_y,\ \tau_{xy}]^{\mathrm{T}}=\boldsymbol{Q}\boldsymbol{\varepsilon}$, where $\boldsymbol{Q}$ is the stiffness coefficient matrix, in which $Q_{11}=Q_{22}=E/(1-\upsilon^2)$, $Q_{12}=Q_{21}=E\upsilon/(1-\upsilon^2)$ and $Q_{66}=E/[2(1+\upsilon)]$, where E and υ are Young's modulus and Poisson's ratio [26].

The equation of motion in each span for the two-span panel is given by [27,28]

$$\frac{\partial^4 w_k}{\partial x_k^4}+2\frac{\partial^4 w_k}{\partial x_k^2\partial y^2}+\frac{\partial^4 w_k}{\partial y^4}+\frac{\rho h}{D}\frac{\partial^2 w_k}{\partial t^2}=0,\quad (k=1,\ 2) \tag{3}$$

where ρ is the mass density of the panel, w_1 and w_2 define the transverse displacements of the left and right sub-panels, and $D=Eh^3/[12(1-\upsilon^2)]$. For the vibration analysis of the two-span panel, the solution of Eq. (3) is assumed as $w_k=W_k(x_k,\ y)\sin(\omega t+\varphi)$, where $W_k(x_k, y)$ is the mode shape of the sub-panel, and ω and φ are the natural frequency and phase angle of the structure. Substitution of the general solution into Eq. (3) yields the following mode equation:

$$\frac{\partial^4 W_k}{\partial x_k^4}+2\frac{\partial^4 W_k}{\partial x_k^2\partial y^2}+\frac{\partial^4 W_k}{\partial y^4}-\alpha^4 W_k=0,\quad (k=1,\ 2) \tag{4}$$

where $\alpha^4=\omega^2\rho h/D$. As far as the fully simply supported two-span panel is concerned, the mode shape $W_k(x_k,\ y)$ can be assumed as

$$W_k(x_k,\ y)=X_k(x_k)\sin(\eta_n y), \tag{5}$$

where $\eta_n=n\pi/b$, n is the number of half wave of the vibration mode in the y direction, and $X(x)$ is the mode shape in the x direction. Substituting Eq. (5) into Eq. (4), the following equation can be obtained:

$$\frac{\partial^4 X_k}{\partial x_k^4}-2\eta_n^2\frac{\partial^2 X_k}{\partial x_k^2}+(\eta_n^4-\alpha^4)X_k=0. \tag{6}$$

The characteristic equation of the above ordinary differential equation is given as

$$\beta_n^4-2\eta_n^2\beta_n^2+(\eta_n^4-\alpha^4)=0, \tag{7}$$

where the subscript "n" indicates that the eigenvalue β is associated with the mode number n in the y direction. The solutions of Eq. (7) can be calculated as

$$\beta_{1n}=\pm\mathrm{i}\sqrt{\alpha^2-\eta_n^2},\quad \beta_{2n}=\pm\sqrt{\alpha^2+\eta_n^2}, \tag{8}$$

where i is the imaginary unit. Consequently, the mode shape of the two-span panel can be expressed as

$$\begin{aligned}W_k(x_k,\ y)=[&A_k\ \sin(\beta_{1n}x_k)+B_k\ \cos(\beta_{1n}x_k)+C_k\ \sinh(\beta_{2n}x_k)\\&+D_k\ \cosh(\beta_{2n}x_k)]\ \sin(\eta_n y),\end{aligned} \tag{9}$$

where A_k, B_k, C_k and D_k are the undetermined coefficients which can be solved according to the boundary conditions of the structure. For the fully simply supported two-span panel, its boundary conditions are given as follows:

$$W_1(0,\ y)=0,\quad M_{x1}(0,\ y)=0, \tag{10a}$$

$$\begin{aligned}&W_1(a_1,\ y)=0,\quad W_2(a_2,\ y)=0,\quad W_{1,x_1}(a_1,\ y)=-W_{2,x_2}(a_2,\ y),\\&M_{x1}(a_1,\ y)=M_{x2}(a_2,\ y),\end{aligned} \tag{10b}$$

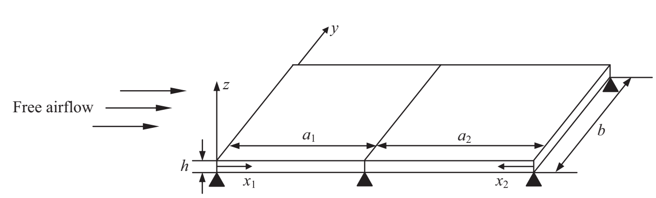

Fig. 1. The schematic diagram of a two-span panel in the supersonic flow.