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Common edge contacts: Effect of interface line orientation

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ABSTRACT

The problem of frictional contact between a finite triangular plane body in frictional contact with an elastically similar frustum, so that the combined geometry is that of a semi-infinite wedge, is studied under plane and anti-plane loading conditions, using a bilateral formulation. The limit of validity of this solution and, in particular the conditions for separation and first slip are found. These results are of relevance to more general contacts having a 'common edge' and suggest whether slip is interior or edge initiated. This is of practical relevance to contacts of this class subject to fretting loading.

1. Introduction

The most common classes of contact arising are incomplete (convex), complete (sharp edged) and receding. But there are other kinds, notably when two components are fastened together which are of the same nominal size, so that the extent of the contact is defined by both bodies simultaneously, and the interface will usually be flat. The object, here, is to investigate an example problem of this kind but where the free surface, while straight, is not, necessarily, perpendicular to the contact interface up to the point of either first slip or separation, whichever occurs first. In the interests of simplicity of definition of the geometry, we will consider a contact formed from a semi-infinite wedge of half angle α, Fig. 1, and where there is a straight interface (shown positioned at a distance d from the apex, although the solution will not, in fact, depend on this distance), and subject to normal and in-plane shear, and possibly anti-plane shear forces, together with an in-plane moment. The last is chosen so as to render the in-plane shear force statically equivalent to one passing along the plane of the contact interface.

Of the classes of contact mentioned, incomplete contacts are unique in that the solution of the traction distribution arising is virtually independent of the geometry of the rest of the contact enabling, for example, the Hertzian contact to be solved with no regard for what the rest of the body looks like. In the case of the complete contacts it is possible to explain quite a lot of what happens near the edges, but a general solution is out of the question, and similar remarks apply here, so that we can hope only to learn general trends.[1] The first two solutions for problems of this kind were (a) that arising when two strips of the same width, with square ends and elastically similar, are pressed end-to-end and subject to shear [1], and (b) the corresponding anti-plane loading problem of two circular bars of the same radius pressed end to end and subject to torsion [2]. These solutions are similar to that to be investigated here when $\alpha \to 0$, and permit us to include just a little more generality in the stress field. We will restrict ourselves to a basic investigation looking at conditions for first slip (and separation), so that use may be made of the 'bilateral' model, where the interface is assumed to be closed and adhered, and a monolithic semi-infinite wedge represents the pair of bodies 'glued' together.

2. Semi-infinite wedge solutions

The classical Flamant solution for a wedge enables us to write down the state of stress present when a normal load, P, and a shear force, Q, are applied at the apex. There is only one non-zero stress component, given by

$$\sigma_{rr}(\theta) = -\frac{P}{r}\frac{2}{2\alpha+\sin 2\alpha}\cos\theta + \frac{Q}{r}\frac{2}{2\alpha-\sin 2\alpha}\sin\theta, \quad \sigma_{r\theta} = \sigma_{\theta\theta} = 0. \tag{1}$$

[1] It should be noted that the progressive contact between a flat indenter with rounded edges and a half-plane can be seen as an intermediate scenario, whereby *only* approximated solutions can be obtained discarding the dependence of the tractions on the detailed indenter geometry. The authors and their co-workers have also discussed this class of contact problems (known also as 'almost complete' contacts) in some of their recent papers.

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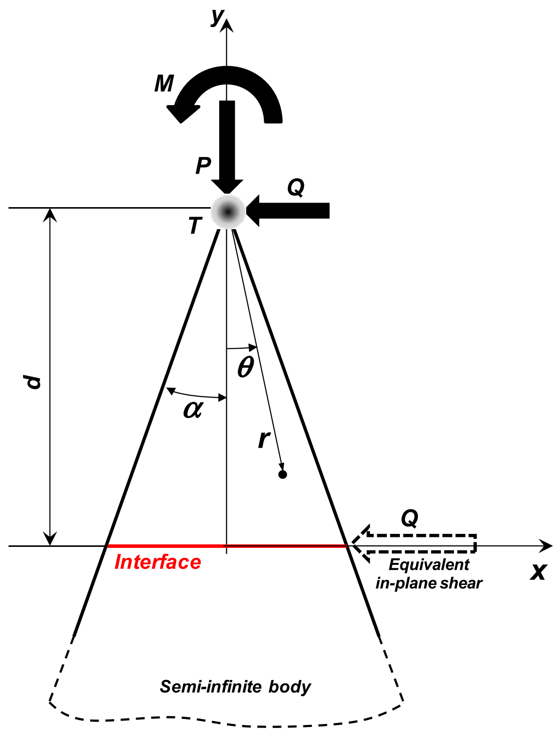

Fig. 1. Geometry of problem showing line of interface (d from wedge apex), tip forces and statically equivalent position of in-plane shear force.

A positive shear force produces positive σ_{rr} in $\theta>0$: it therefore acts in the negative x direction, and a positive normal load acts in the negative y direction, Fig. 1. The application of a moment, M, produces a stress distribution given by (see Barber [3])

$$\frac{\sigma_{rr}}{M}=-\frac{2\,\sin(2\theta)}{r^2[2\alpha\,\cos\,2\alpha-\sin\,2\alpha]} \tag{2}$$

$$\frac{\sigma_{r\theta}}{M}=-\frac{\cos\,2\alpha-\cos(2\theta)}{r^2[2\alpha\,\cos\,2\alpha-\sin\,2\alpha]}. \tag{3}$$

$$\sigma_{\theta\theta}=0. \tag{4}$$

A positive moment produces positive σ_{rr} in $\theta>0$: it therefore acts in an anticlockwise direction, Fig. 1. Suppose we introduce a straight interface at a distance, d, from the wedge apex. In order to make the shear force, Q, statically equivalent to one passing along the interface we set

$$M=-Qd \tag{5}$$

and introduce a new x-axis which is straight and perpendicular to the θ=0 radial line. Hence we can write down the state of stress along this line as

$$\sigma_{rr}(x)=-\frac{Pd}{r^2}A_1+\frac{Qx}{r^2}\left[A_2+\frac{4d^2}{d^2+x^2}A_3\right]$$
$$-\tan\,\alpha\le\frac{x}{d}\le\tan\,\alpha \tag{6}$$

$$\sigma_{r\theta}(x)=\frac{Qd}{r^2}\left\{A_4-\frac{d^2-x^2}{d^2+x^2}A_3\right\} \tag{7}$$

where

$$r^2=d^2+x^2 \tag{8}$$

$$\tan\,\theta=\frac{x}{d};\quad \cos\,\theta=\frac{d}{r};\quad \sin\,\theta=\frac{x}{r}, \tag{9}$$

and the coefficients (functions of α) are

$$A_1=\frac{2}{2\alpha+\sin\,2\alpha} \tag{10}$$

$$A_2=\frac{2}{2\alpha-\sin\,2\alpha} \tag{11}$$

$$A_3=\frac{1}{[2\alpha\,\cos\,2\alpha-\sin\,2\alpha]} \tag{12}$$

$$A_4=\frac{\cos\,2\alpha}{[2\alpha\,\cos\,2\alpha-\sin\,2\alpha]}. \tag{13}$$

The next step is to transform the state of stress into Cartesian coordinates by rotating anticlockwise by $(\pi/2-\theta)$, i.e. $\theta\to y, r\to x$, giving

$$\sigma_{yy}(x)=-\frac{Pd^3}{(d^2+x^2)^2}A_1+\frac{Qxd^2}{(d^2+x^2)^2}\left[A_2+\frac{6d^2-2x^2}{d^2+x^2}A_3-2A_4\right] \tag{14}$$

$$\tau_{xy}(x)=\frac{Pxd^2}{(d^2+x^2)^2}A_1-\frac{Qd}{(d^2+x^2)^2}\left[x^2A_2+-\frac{(d^2-x^2)^2-4x^2d^2}{d^2+x^2}A_3+(d^2-x^2)A_4\right] \tag{15}$$

This is the basic solution and, although the absolute value of the state of stress depends on the depth of the interface, its qualities are independent of d, and we now probe violations of the Signorini contact conditions.

First, the point where separation is most likely is at $\theta=\alpha$, $x/d=\tan\,\alpha$, and occurs when

$$\frac{Q}{P}=\frac{A_1\cot\alpha}{\left[A_2+\frac{4}{1+\tan^2\alpha}A_3\right]-2\left\{A_4-\frac{1-\tan^2\alpha}{1+\tan^2\alpha}A_3\right\}}$$
$$=\frac{A_1(1+\tan^2\alpha)\cot\alpha}{A_2+6A_3-2A_4+(A_2-2A_3-2A_4)\tan^2\alpha}.$$

Secondly, the traction ratio is given by

$$\frac{\tau_{xy}}{\sigma_{yy}}(x)=\frac{-\left\{-A_1+\frac{Qx}{Pd}\left[A_2-\frac{4d^2}{d^2+x^2}A_3\right]\right\}\frac{xd}{d^2+x^2}-\frac{Q}{P}\left\{A_4-\frac{d^2-x^2}{d^2+x^2}A_3\right\}\frac{d^2-x^2}{d^2+x^2}}{\left\{-A_1+\frac{Qx}{Pd}\left[A_2-\frac{4d^2}{d^2+x^2}A_3\right]\right\}\frac{d^2}{d^2+x^2}-2\frac{Q}{P}\left\{A_4-\frac{d^2-x^2}{d^2+x^2}A_3\right\}\frac{xd}{d^2+x^2}} \tag{16}$$

so that we can easily find the minimum coefficient of friction to ensure adhesion at all points along the interface, by ensuring that it is at least equal to the maximum value of this ratio.

At a free surface, as both tractions vanish, the only surviving stress component is the direct stress (σ_0 say) parallel with the free edge. We can easily transform this into adjacent tractions present along the interface giving

$$\sigma_{yy}=\tfrac{1}{2}\sigma_0(1+\cos\,2\alpha) \tag{17}$$

$$\tau_{xy}=\tfrac{1}{2}\sigma_0\,\sin\,2\alpha. \tag{18}$$

So, the minimum coefficient of friction to ensure full adhesion at a contact edge is given by

$$f>\frac{\sin\,2\alpha}{(1+\cos\,2\alpha)}=\tan\,\alpha. \tag{19}$$

This is a general result for in-plane loading and is independent of the loading conditions.

3. Anti-plane and combined loading

In the torsion problem [2] the first point to slip is always the edge of the contact disk, but, of course, the shear stress increases linearly with radius in that problem. Suppose that, here,