International Journal of Mechanical Sciences 81 (2014) 88–94

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

A new trigonometric beam model for buckling of strain gradient microbeams

Bekir Akgöz, Ömer Civalek*

Akdeniz University, Civil Engineering Department, Division of Mechanics, Antalya, Turkiye

ARTICLE INFO

Article history:
Received 21 May 2013
Received in revised form
3 February 2014
Accepted 11 February 2014
Available online 18 February 2014

Keywords:
Size effect
Microbeam
Modified strain gradient theory
Buckling
Trigonometric shear deformation theory

ABSTRACT

In this paper, a new microstructure-dependent sinusoidal beam model for buckling of microbeams is presented using modified strain gradient theory. This microbeam model can take into consideration microstructural and shear deformation effects. The equilibrium equations and corresponding boundary conditions in buckling are derived with the minimum total potential energy principle. Buckling problem of a simply supported microbeam subjected to an axial compressive force is analytically solved by Navier solution procedure. Influences of thickness-to-length scale parameter and slenderness ratios on buckling behavior are discussed in detail. It is observed that the size dependency becomes more important when the thickness of the microbeam is closer to material length scale parameter. Also, it can be said that the effects of shear deformation are more considerable for short and thick beams with lower slenderness ratios.

1. Introduction

Micro-sized structural and mechanical elements like bar, beam, plate and shell are considered as frequently used basic components of many micro-electro mechanical systems (MEMS) [1–5]. The characteristic dimensions of these elements are typically on the order of microns and sub-microns. It has been reported in some experimental studies that the structures become stiffer in smaller sizes [6–8]. In the absence of any intrinsic or length scale parameters, classical continuum theories do not have the ability to predict the microstructure-dependent deformation behavior of micro- and nano-sized structures. In order to determine the mechanical responses of such structures, several non-classical continuum theories have been developed such as couple stress theory [9–11], micropolar theory [12], nonlocal elasticity theory [13,14] and strain gradient theories [15–18].

The modified strain gradient theory was proposed by Lam et al. [7] in which there is an additional equilibrium equation of moments of couples besides the well-known classical equilibrium equations of forces and moments of forces. This modified theory has been employed to develop size-dependent beam models. For instance, Kong et al. [19] and Wang et al. [20] developed Bernoulli–Euler and Timoshenko microbeam models for bending and vibration responses, respectively. Buckling and bending analysis of Bernoulli–Euler microbeams with various boundary conditions was also carried out by present authors [21,22]. Furthermore, Kahrobaiyan et al. [23], Akgoz and Civalek [24] and Ansari et al. [25] introduced Bernoulli–Euler and Timoshenko beam models for inhomogeneous functionally graded microbeams, respectively. Also, this theory has been utilized to formulate microbars [26–31] and nonlinear microbeam models [32–35].

For linear elastic isotropic materials, modified strain gradient theory contains three additional material length scale parameters relevant to dilatation gradients (l_0), deviatoric stretch gradients (l_1) and rotation gradients (l_2), respectively. It is notable that if dilatation gradients and deviatoric stretch gradients are omitted ($l_0 = l_1 = 0$), the formulation and governing equations of this theory will be transformed to those of modified couple stress theory proposed by Yang et al. [36]. This simpler theory has been utilized to investigate static and dynamic responses of size-dependent microbeams [37–46]. Furthermore, thermal effect on buckling and free vibration responses of functionally graded microbeams based on this theory was investigated by Nateghi and Salamat-talab [47].

By this time, several beam theories have been proposed by many researchers. The well-known of them are Euler–Bernoulli (EBT) and Timoshenko (TBT) beam theories. According to assumptions in EBT, effects of shear deformation are ignored. The use of this theory can be suitable for slender beams with a large aspect ratio. However, effects of shear deformation can be more prominent for moderately thick beams. TBT is an earlier shear deformation beam theory. TBT assumes that transverse shear stress and

* Corresponding author. Tel.: +90 242 3106319; fax: +90 242 3106306.
E-mail address: civalek@yahoo.com (Ö. Civalek).

http://dx.doi.org/10.1016/j.ijmecsci.2014.02.013

strain are invariant throughout the thickness of the beam. However, there are no transverse shear stress and strain at the upper and lower surfaces of the beam and their distributions are not uniform. For this reason, a shear correction factor is needed in formulation. After that, some higher-order shear deformation beam theories, which satisfy the condition of no shear stress and strain at the top and bottom surfaces of the beam without any shear correction factors, have been presented such as parabolic (third-order) beam theory [48,49], trigonometric (sinusoidal) beam theory [50], hyperbolic beam theory [51], exponential beam theory [52] and a general exponential beam theory [53]. Static and dynamic analyses of nonhomogeneous beams have been investigated on the basis of various higher-order shear deformable beam theories [54–56]. Also, several size-dependent beam models have been developed on the basis of aforementioned beam theories in conjunction with nonlocal elasticity theory [57–61], modified couple stress and strain gradient theories [62–67].

The purpose of this study is to introduce a new non-classical sinusoidal shear deformation beam model on the basis of modified strain gradient theory in order to investigate stability response of microbeams. The equilibrium equations and corresponding boundary conditions in buckling are derived with the aid of minimum total potential energy principle. Buckling problem of a simply supported microbeam subjected to an axial compressive force is analytically solved by Navier solution procedure. A detailed parametric study is performed to show the influences of thickness-to-length scale parameter ratio and slenderness ratio on buckling behavior of microbeams. The effects of shear deformation can be considerable for short beams with lower aspect ratios and a shear deformation beam theory should be used in modeling and analysis.

2. Theory and formulation

According to the modified strain gradient elasticity theory, strain energy U can be written with infinitesimal deformations as [7]

$$U=\frac{1}{2}\int_0^L\int_A(\sigma_{ij}\varepsilon_{ij}+p_i\gamma_i+\tau^{(1)}_{ijk}\eta^{(1)}_{ijk}+m^s_{ij}\chi^s_{ij})dA\,dx \tag{1}$$

$$\varepsilon_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i}) \tag{2}$$

$$\gamma_i=\varepsilon_{mm,i} \tag{3}$$

$$\eta^{(1)}_{ijk}=\frac{1}{3}(\varepsilon_{jk,i}+\varepsilon_{ki,j}+\varepsilon_{ij,k}) -\frac{1}{15}[\delta_{ij}(\varepsilon_{mm,k}+2\varepsilon_{mk,m})+\delta_{jk}(\varepsilon_{mm,i}+2\varepsilon_{mi,m})+\delta_{ki}(\varepsilon_{mm,j}+2\varepsilon_{mj,m})] \tag{4}$$

$$\chi^s_{ij}=\frac{1}{2}(\theta_{i,j}+\theta_{j,i}) \tag{5}$$

$$\theta_i=\frac{1}{2}e_{ijk}u_{k,j} \tag{6}$$

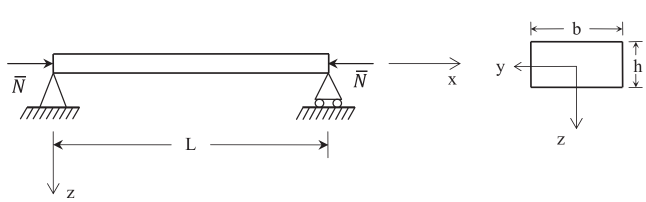

Fig. 1. Geometry and cross-section of a simply supported beam subjected to an axial compressive force.

where u_i, θ_i, ε_{ij}, γ_i, $\eta^{(1)}_{ijk}$ and χ^s_{ij} denote the components of the displacement vector $\mathbf{u}$, the rotation vector $\boldsymbol{\theta}$, the strain tensor $\boldsymbol{\varepsilon}$, the dilatation gradient vector $\boldsymbol{\gamma}$, the deviatoric stretch gradient tensor $\boldsymbol{\eta}^{(1)}$ and the symmetric rotation gradient tensor $\boldsymbol{\chi}^s$, respectively. Also, δ and e_{ijk} are the Kronecker delta and the alternating symbols, respectively.

Furthermore, the components of the classical and higher-order stress tensors are expressed as following [7]:

$$\sigma_{ij}=\lambda\varepsilon_{mm}\delta_{ij}+2\mu\varepsilon_{ij} \tag{7}$$

$$p_i=2\mu l_0^2\gamma_i \tag{8}$$

$$\tau^{(1)}_{ijk}=2\mu l_1^2\eta^{(1)}_{ijk} \tag{9}$$

$$m^s_{ij}=2\mu l_2^2\chi^s_{ij} \tag{10}$$

where l_0, l_1, and l_2 are additional material length scale parameters relevant to dilatation gradients, deviatoric stretch gradients and rotation gradients, respectively. Furthermore, λ and μ are the Lamé constants defined as

$$\lambda=\frac{Ev}{(1+v)(1-2v)},\qquad \mu=\frac{E}{2(1+v)} \tag{11}$$

The displacement components of an initially straight beam (see Fig. 1) on the basis of sinusoidal beam theory (SBT) can be written as [50]

$$\begin{aligned}u_1(x,z)&=u(x)-z\frac{dw(x)}{dx}+R(z)\phi(x)\\ u_2(x,z)&=0\\ u_3(x,z)&=w(x)\end{aligned} \tag{12}$$

in which

$$\phi(x)=\frac{dw(x)}{dx}-\varphi(x) \tag{13}$$

where u_1, u_2 and u_3 are the x-, y- and z-components of the displacement vector, and also u and w are the axial and transverse displacements, respectively, φ is the angle of rotation of the cross-sections about y-axis of any point on the mid-plane of the beam, respectively. $R(z)$ is a function which depends on z and plays a role in determination of the transverse shear strain and stress distribution throughout the height of the beam. In order to satisfy no shear stress and strain condition at the upper and lower surfaces of the beam, $R(z)$ is selected as following without need for any shear correction factors:

$$R(z)=\frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right) \tag{14}$$

It can be noted that the displacement components for EBT, TBT and parabolic (third-order) beam theory (PBT) will be obtained by setting $R(z)=\{0,z,z(1-4z^2/3h^2)\}$, respectively. Using Eqs. (12)–(14) into Eq. (2), we obtain the non-zero strain components as

$$\varepsilon_{11}=u'-zw''+R\phi',\qquad \varepsilon_{13}=\frac{1}{2}R_{,z}\phi \tag{15}$$

where

$$u'=\frac{du}{dx},\quad \phi'=\frac{d\phi}{dx},\quad w''=\frac{d^2w}{dx^2},\quad R_{,z}=\frac{dR(z)}{dz} \tag{16}$$

and by using above equations in Eqs. (3)–(5), the non-zero components of higher-order gradients are expressed as

$$\gamma_1=u''-zw'''+R\phi'',\qquad \gamma_3=-w''+R_{,z}\phi' \tag{17}$$

$$\eta^{(1)}_{111}=\frac{1}{5}[2(u''-zw'''+R\phi'')-R_{,zz}\phi],$$
$$\eta^{(1)}_{113}=\eta^{(1)}_{131}=\eta^{(1)}_{311}=-\frac{4}{15}(w''-2R_{,z}\phi'),$$