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Mechanics of anisotropic hierarchical honeycombs

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ABSTRACT

Anisotropic hierarchical honeycombs of uniform wall-thickness are constructed by repeatedly replacing each three-edge vertex of a base hexagonal network with a similar but smaller hexagon of the same orientation, and stretching the resulting structure in horizontal or vertical directions to break the isotropy. The uniform overall thickness is then adjusted to maintain the constant average density. The resulting fractal-appearing hierarchical structure is defined by the ratios of replacement edge lengths to the underlying network edge length and also the cell wall angle. The effective elastic modulus, Poisson's ratio and plastic collapse strength in the principal directions of hierarchical honeycombs were obtained analytically as well as by finite element analyses. The results show that anisotropic hierarchical honeycombs of first to fourth order can be 2.0–8.0 times stiffer and at the same time up to 2.0 times stronger than regular honeycomb at the same wall angle and the same overall average density. Plastic collapse analysis showed that anisotropic hierarchical honeycomb has the larger plastic collapse strength compared to regular hierarchical honeycomb of the same order at certain oblique wall angles. The current work provides insight into how incorporating anisotropy into the structural organization can play a significant role in improving the mechanics of the materials structure such as regular or hierarchical honeycombs, and introduces new opportunities for development of novel materials and structures with desirable and actively tailorable properties.

1. Introduction

Materials with structural hierarchy over nanometer to millimeter length scales are found throughout Kingdoms *Plantae* and *Animalia*. Examples include bones and teeth [1,2], nacre (mother-of-pearl) [3], gecko foot pads [4], Asteriscus (yellow sea daisy) [5], *Euplectella sponge* [6], wood [2,7] and water repellent biological systems [8]. The idea of using structural hierarchy in engineering structures and materials goes back at least to Eiffel's Garabit Viaduct and then Tower [9]. More modern examples include polymers [10], composite structures [11–13] and sandwich panel cores [14,15]. The effect of structural hierarchy on mechanical and chemical properties of biological and biomimetic systems has been extensively documented [9–17]. The type and order of the hierarchy and the general organization of these structures play a significant role in their properties and functionality [16,17]. For example, Zhang et al. [16] showed that increasing the level of hierarchy in biological materials increases the toughness but decreases the strength, suggesting that an optimal level of hierarchy could be defined.

Incorporating hierarchy into honeycomb lattice structures has been the focus of a number of studies [15,18–21] and has significance with regard to the application of honeycombs in impact energy absorption and structural protection [22–26], thermal isolation [27] and as the structural core of sandwich panels [28–32]. Recently, a new generation of honeycombs with hierarchical organization was achieved by replacing nodes in the regular honeycomb with smaller hexagons [18,19]. One or two orders of optimized hierarchical refinement offered up to 2 and 3.5 times the in-plane stiffness [18] and almost 2 times the plastic collapse strength [19,20] of conventional honeycomb with the same mass. Majority of these works address the mechanical and thermal properties of isotropic honeycomb structures. However, there are relatively little investigations on the mechanical properties of honeycomb structure with stretched cells resulting in anisotropic honeycomb structures. The present paper extends these previous works by horizontally/vertically 'stretching' or reformulating the underlying hexagonal network prior to the hierarchical refinement steps, so that the developed structure is no longer isotropic (wall thickness is maintained uniform, while being adjusted to have fix overall average density as hierarchy is introduced).

In this work, anisotropic hierarchical honeycombs with various oblique-wall angles are compared to hierarchical conventional honeycombs (with $\theta = 30^{o}$). The stretches not only alter the cell

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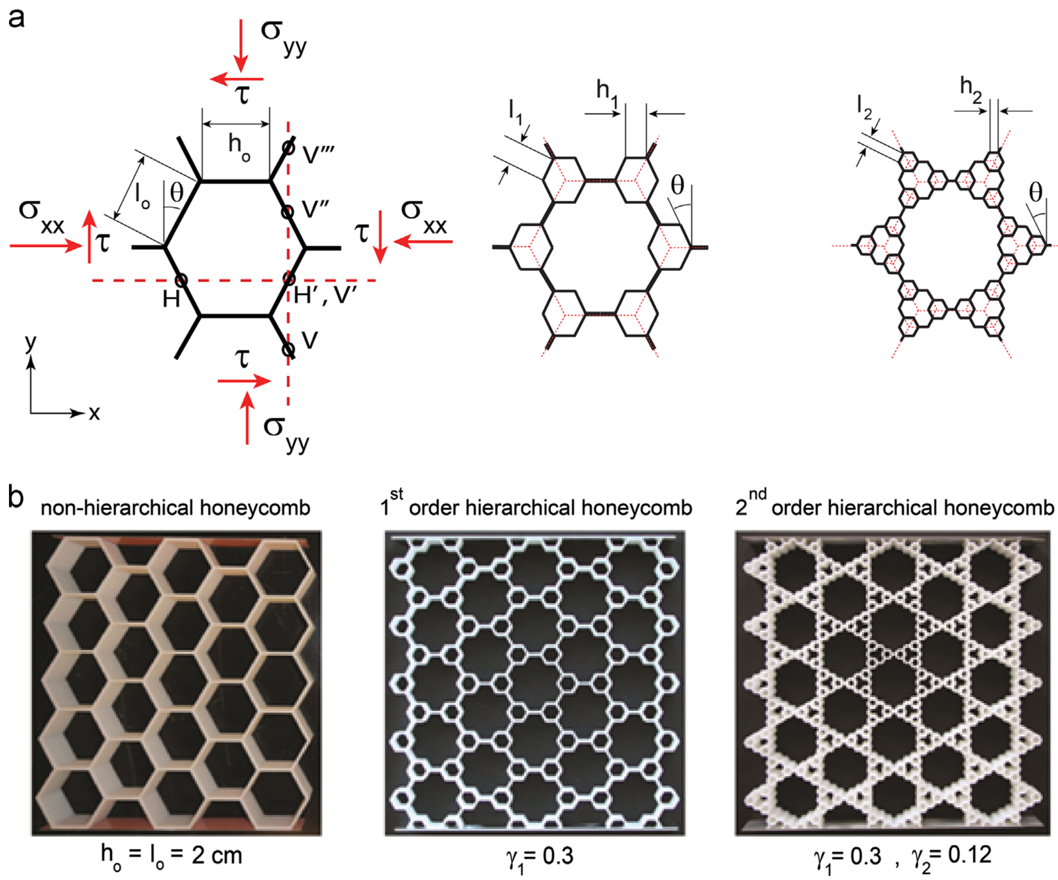

Fig. 1. (a) Sections of the non-hierarchical honeycomb structure (left) and honeycomb structures with one (middle) and two (right) orders of hierarchy. In order for the intersections of newly generated hexagons lie on the edge of previous hexagons, $h_i = 2l_i \sin(\theta)$ for $i = 0, 1, 2, \ldots$ should be satisfied. (b) Images of regular honeycombs with $l_0 = h_0 = 2$ cm fabricated using three-dimensional printing ((b) is taken from [18]).

wall lengths, but it also changes the oblique wall angle, θ (which is equal to 30° in the conventional isotropic honeycomb). Note that uniform stretch leaves oblique cell walls still pointing at the centers of hexagons above and below. Since equal vertical and horizontal stretches would leave the hexagonal geometry undistorted, we 'normalize' the transformation: the length of an oblique cell wall was taken as fixed, while its angle is selected within the range $0 < \theta < \pi/2$. The distorted hexagons of the underlying network therefore have height $2l_0 cos(\theta)$ and horizontal edge length $2l_0 sin(\theta)$ (Fig. 1a). In hierarchically refined structures of uniform thickness, the structural organization is uniquely defined by the ratio of the introduced oblique edge lengths (l_1 and l_2, respectively, for first and second order of hierarchy) to the original hexagon's oblique edge length, l_0. These are denoted $\gamma_1 = l_1/l_0$ and $\gamma_2 = l_2/l_0$, etc., where l_0, l_1, l_2 are defined in Fig. 1a. At each order of hierarchy the introduced *horizontal* edge length conforms to $h_i = 2l_i sin(\theta)$, where l_i is the introduced oblique edge length, θ is the oblique wall angle and h_i is the introduced horizontal edge length.

Here we explore up to four orders of hierarchy. The elastic properties of first and second order hierarchy are evaluated theoretically by Castigliano's method and compared to a matrix frame analysis carried out in MATLAB. The elastic moduli of third and fourth order hierarchy are therefore evaluated numerically only. In Section 2, fabrication of samples using 3D printing is outlined. In Section 3, elastic properties of anisotropic hierarchical honeycombs using Castigliano's second theorem are determined. In section 4, the numerical analysis which carried out in MATLAB is outlined. In Section 5, the effective plastic collapse strength for uniaxial in-plane loading in principal directions is determined using elastic–plastic beam elements in the finite element package ANSYS. In Section 6, results and discussion are demonstrated. In Section 7, conclusions and potential for further performance improvement of hierarchical honeycombs are presented.

2. Fabrication using 3D printing

Fig. 1b shows samples of both zero order and hierarchical *regular*-hexagon honeycombs with relative density of $\overline{\rho} = \rho/\rho_s = 0.10$ and $l_0 = 20$ mm, where ρ is the structural density and l_0 is the oblique hexagon edge length [18]. These samples were fabricated using 3D printing (Dimensions 3D printer, Stratasys Inc., Eden Prairie, MN). The regular honeycomb has $t = 1.75$ mm; the honeycomb with one level of hierarchy has $\gamma_1 = 0.3$ and $t = 1$ mm; and that with two-level hierarchy has $\gamma_1 = 0.3$, $\gamma_2 = 0.12$, and $t = 0.75$ mm, where t is the hexagons wall thickness. These were printed as three-dimensional extruded shells from an ABS polymer (acrylonitrile butadiene styrene, elastic modulus = 2.3 GPa). Fig. 2a shows the images of 1st order anisotropic honeycombs with $\theta = 10^o$, 30^o and 70^o and $l_0 = 2$ cm fabricated using three-dimensional printing. All three structures have $\gamma_1 = l_1/l_0 = 0.3$.

3. Elastic properties of anisotropic hierarchical honeycombs: analytical modeling

Some geometric constraints on the hierarchically introduced edges must be imposed to avoid interfering with pre-existing members: in a honeycomb with first order hierarchy, $0 \le l_1 \le l_0/2$ (Fig. 1a) and thus, $0 \le \gamma_1 \le 0.5$, where $\gamma_1 = 0$ denotes the regular honeycomb structure. For second order hierarchy, the two geometrical constraints are $0 \le l_2 \le l_1$ and $l_2 \le l_0/2 - l_1$. For uniform wall