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Thermal buckling of shear deformable temperature dependent circular/annular FGM plates

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ABSTRACT

Bifurcation behaviour of moderately thick heated annular plates made of FGMs is discussed in this study. Properties of the graded plate are distributed across the thickness based on the simple power law form. Temperature-dependency of the material properties is also taken into account. Two types of frequently used thermal loading, i.e. uniform temperature rise and heat conduction across the thickness loadings are considered. General nonlinear equilibrium equations based on the first order shear deformation plate theory (FSDT) and the associated boundary conditions are obtained employing the static version of the virtual displacements method. The pre-buckling solution is accomplished and the proper boundary conditions are chosen to assure the occurrence of bifurcation phenomenon. Stability equations are obtained based on the adjacent equilibrium criterion. Five resulted stability equations are decoupled and reduced to new equations in terms of lateral deflection and the edge zone functions. An exact asymmetrical solution is developed to calculate the critical buckling temperature difference of the plate for the above-mentioned cases of thermal loading. It is shown that the fundamental buckled configuration of annular plates may be asymmetric as previously assumed.

1. Introduction

Functionally graded materials are known as a class of novel materials in which properties are graded in one, two or even three specific direction(s). Interesting behaviour of solid structures made of FGMs have attracted the researchers attention, in recent years. Consequently, a large number of investigations in the field of structural stability refers to the analysis of FGM solids. Especially, circular, annular or sectoral plates which are known as the primary elements of various complicated structures have revealed the extensive applications in structural, mechanical, civil and aerospace engineering.

Primary researches in the field of structural stability of FGM solids in polar coordinate belong to Najafizadeh and Eslami [1,2]. In these investigations, the general equilibrium and stability equations are extracted. The eigen-value analysis of axisymmetric stability equations is accomplished to detect the critical buckling temperature as well as critical buckling loads of through-the-thickness FGM solid circular plates. It is shown that a closed-form expression may be used to predict the critical buckling states of the plate. This formula is the same with the one which is used for homogeneous plates. However, the equivalent flexural rigidity of the FGM plate, in which the extensional-bending coupling is included, should be used instead of the one belongs to the homogeneous plates. Motivated by the above-mentioned works, Najafizadeh and Heydari [3,4] studied the axisymmetric thermal and mechanical buckling of thick circular plates made of FGMs within the framework of third order shear deformation plate theory of Reddy. Results of these researches reveal that thermal buckling phenomenon may not happen for thick FGM plates under the most practical cases of thermal loadings. For other investigations on axisymmetric thermal buckling of solid circular plates one may refer to [5–7].

Jalali et al. [8] carried out the thermal buckling analysis of sandwich plates having nonuniform thickness incorporated with FGM face sheets. Unlike the case of plates with uniform thickness, non-uniformity of the thickness results in the non-uniform radial deformation in the pre-buckling state. Consequently, a shooting method is implemented to study the pre-buckling deformation of the plate. Pre-buckling deformations are inserted into the axisymmetric stability equations and the eigen-value solution is accomplished by means of a pseudo-spectral method. Most recently, Kiani and Eslami [9] studied the influence of the partial elastic foundation on the buckling patterns and bifurcation temperatures of FGM solid circular plates. The study concluded that,

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in some cases of foundation stiffness and contact radius, asymmetric buckling patterns are associated to the fundamental buckling temperatures of a symmetrically heated clamped FGM plate. There are also some researches in which critical buckling temperatures are extracted through the complete nonlinear post-buckling equilibrium path. In this case, nonlinear equilibrium equations are solved rather than the linearised stability equations. Researches of Ma and Wang based on shooting method [10], Li et al. based on shooting method [11] and Fallah and Nosier based on two-step perturbation scheme [12] are some of the available investigations in this category.

There are also a few investigations on the thermal and mechanical buckling of through-the-thickness FGM plates in a sectoral shape. Saidi and his co-authors [13–15] examined the thermal buckling of sectoral plates [13], mechanical buckling of uniformly compressed sector plates [14], and the influence of uniform Pasternak elastic foundation on mechanical stability resistance of FGM sector plates [15]. In all of these works, first order plate theory is implemented to derive the governing equilibrium equations in the most general form. Linear pre-buckling analysis is performed and the resulting stability equations are decoupled in terms of lateral displacement function and a new function known as the boundary layer function. These researches cover the Levy-type of boundary conditions for sectoral plate. However, considering the bifurcation-type buckling for through-the-thickness FGM plates with at least one edge simply supported is not the real state of the plate.

Stability analysis of FGM plates in annular shape is almost rare in the open literature. Ma and Wang [16] employed the shooting method to study the axisymmetric thermal buckling of annular plates. Ghomshei and Abbasi [17] investigated the axisymmetric thermal buckling of annular plates with non-uniform thickness based on a finite elements method. Sepahi et al. [18] analysed the axisymmetric buckling and post-buckling of radially graded FGM plates subjected to radial heat conduction. Due to the distribution of thermo-mechanical properties through the radial direction, non-uniform radial displacements are generated through the plate domain. One-dimensional generalised differential quadrature (GDQ) is used to study the pre-buckling and the post-buckling deflection of the plate. In this investigation, temperature dependency of the material properties is also taken into account. In another study, Aghelinejad et al. [19] applied the shooting method to examine the post-buckling deflection of a through-the-thickness graded, radially heated plate. Only the case of a plate with both edges clamped is analysed in this research. Most recently, Kiani and Eslami [20] studied the existence of unsymmetrical buckling pattern in a symmetrically heated annular plate. Only the case of a plate with both inner and outer edges clamped is examined. It is shown that, for both contact-less and in-contact annular FGM plates, the fundamental buckling temperature of the plate is associated to an asymmetrical buckling pattern. Consequently, in some cases, when stability equations of a symmetrically heated annular plate are solved symmetrically, critical buckling temperatures are overestimated and buckling configurations are wrongly depicted.

Motivated by previous research of Kiani and Eslami [20], present study extends the analysis of [20] from the thin temperature independent plates to the shear deformable temperature dependent plates. Various cases of edge supports are taken into account. Properties of the constituents are both position and temperature dependent. Dependency of the constituents to the temperature and position are described in terms of, respectively, Touloukian model and a simple power law function. The complete set of asymmetrical equilibrium equations are established based on the static version of virtual displacements. The resulted equations are uncoupled and solved exactly. Two types of thermal

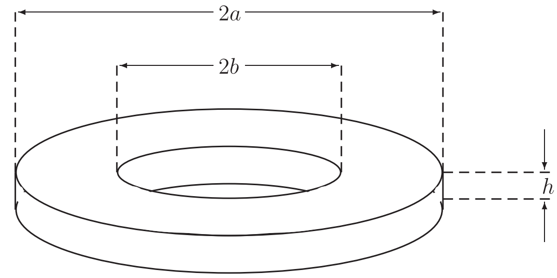

Fig. 1. Geometry of a functionally graded annular plate.

loading are considered. Closed-form solutions are presented to predict the critical buckling temperature differences in each case. It is shown that temperature dependency of the constituents, inner and outer radii of the plate, edge support conditions and power law index are influential parameters in estimation of buckled pattern and buckling temperatures of heated plates.

2. Fundamental equations of FG annular plate

Consider an annular plate made of FGMs of thickness h, inner radius b and outer radius a, as shown in Fig. 1. Polar coordinate system (r, θ, z), with its origin located at the centre of the plate mid-surface is defined. In this system, r, θ and z represent, respectively, the radial, circumferential and through-the-thickness directions.

2.1. Material properties of FGMs

Equivalent properties of the FGM plate should be defined according to a proper homogenization method. Voigt rule is commonly used for this reason. According to this rule, the mechanical and thermal properties of the FGM plate, such as Young's modulus E, Poisson's ratio ν, thermal expansion coefficient α, and thermal conductivity K are assumed as the linear function of the volume fractions of the ceramic V_c and metal V_m. Thus, as a function of thickness direction, a non-homogeneous property of the plate, P, may be expressed in the form [9,20,21]

$$P(z,T) = P_m(T) + V_c(z)P_{cm}(T),\ \ P_{cm}(T) = P_c(T) - P_m(T) \tag{1}$$

where the subscripts m and c represent the properties of metal and ceramic constituents, respectively. Temperature dependency of the FGM constituents are frequently expressed based on Touloukian formula [22]. Accordingly, each property of the metal or ceramic may be written in the form

$$P(T) = P_0(P_{-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3) \tag{2}$$

in which T is temperature measured in Kelvin and P_i's are temperature-dependence coefficients, unique to the constituents. Following Reddy, a simple power law function may be used to represent the ceramic volume fraction, V_c and metal volume fraction, V_m such as bellow [22]

$$V_c = \left(\frac{1}{2} + \frac{z}{h}\right)^k, V_m = 1 - V_c \tag{3}$$

Here k is a non-negative constant called the power law index and dictates the property dispersion profile.

2.2. Kinematic assumptions

Displacement field through the plate domain is assumed to obey the first order shear deformation plate theory (*FSDT*). Based on the *FSDT*, the displacement components of the plate may be