International Journal of Mechanical Sciences 81 (2014) 158–164

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Analysis of graded coatings for resistance to contact deformation and damage based on a new multi-layer model

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ARTICLE INFO

Article history:
Received 14 August 2013
Received in revised form
24 January 2014
Accepted 10 February 2014
Available online 19 February 2014

Keywords:
Contact
Functionally graded coating
Multi-layer model
The transfer matrix

ABSTRACT

The contact-damage resistance of functionally graded materials coating is considered in this paper. We build a new multi-layer computational model to simulate the functionally graded materials with arbitrary spatial variation of material properties with no limit to Poisson's ratio. In this model, the graded coating is divided into several sub-layers with the elastic modulus varying as exponential function and Poisson's ratio is a constant, according to a curve can be approached by a series of continuous but piecewise exponential function. The axisymmetric contact problem is formulated in terms of a singular integral equation by using the transfer matrix and the Hankel integral transform technique. It is assumed that the shear modulus of graded coating varies as the power law forms along the thickness and that the graded coating is indented by a rigid spherical indenter. The effect of the variation of Poisson's ratio, the gradient index and the value of Poisson's ratio on the contact stress, contact radius and penetration depth are calculated by solving an equation numerically.

1. Introduction

The coating–substrate system has been widely applied in engineering practices, in order to improve the reliability and durability of components [1]. Traditionally, the homogeneous coating includes the discontinuity of material properties at the interface which result in stress concentration, crack initiation or delaminating at the interfaces [2]. To overcome this disadvantage, the functionally graded materials (FGMs) have been explored as an alternative to the conventional homogenous coatings. Recently, Suresh [3] proved that FGM coatings can resist contact deformation and damage by altering the gradients of materials. Since then the contact problem for FGMs has received much attention, and many interesting researches have been reported. Because FGMs are composites whose material properties vary gradually along a coordinate axis, it is very difficult to solve the governing equations which represent the mechanical behaviors of the materials. Some specific functional forms such as exponential functions and power-law functions of elastic modulus have been assumed to describe the properties of FGMs. Guler and Erdogan [4–6] applied the singular integrate equation method to solve the two-dimensional contact problem of functionally graded coatings. They assumed the materials properties vary as exponential functions. Jeon et al. [7] investigated axisymmetric problems for a non-homogeneous elastic layer subjected to an arbitrarily shaped distributed loading on its upper and lower surfaces. They assumed the shear modulus of the elastic layer varies by the power product form. The axisymmetric frictionless contact problem of the functionally graded coatings with exponentially varying modulus was considered by Liu and Wang [8]. In order to simulate FGMs with arbitrary variation of the material properties, laminated composite structure [9], which is formed by homogeneous laminas with different properties bonding together, is used. Because laminated composite involves discontinuities of the material properties at the sub-interfaces, it can result in crack initiation and interface debonding. Recently, Ke and Wang [10–13] constructed a linear multi-layered model for FGMs which allows arbitrary variation of the material properties to solve the two-dimensional contact problem. Liu et al. [14–16] extended the linear multi-layered model to the three-dimensional axisymmetric contact problem of a functionally graded coated half-space. In their research, the shear modules of FGM coatings can be described by arbitrary function, but Poisson's ratio is limited to 1/3.

In the present paper, a new multi-layer computational model will be built to simulate the functionally graded materials with arbitrary spatial variation of material properties with no limit to Poisson's ratio. In this model, an arbitrary curve may be approached by a series of continuous but piecewise exponential function. The Hankel integral transform technique and transfer matrix method are used to solve the axisymmetric contact problem of graded coating–substrate structure. The shear modulus of graded coatings is assumed to vary as power law form. The contact pressure, the relation of force–indentation and the relation of force–contact radius are presented for the various values of parameters representing the variation of

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http://dx.doi.org/10.1016/j.ijmecsci.2014.02.009
0020-7403

Poisson's ratio, the gradient index, and the value of Poisson's ratio, calculated by solving the equation numerically.

2. A new multi-layer model to simulate the functionally graded materials

Fig. 1 shows the arbitrarily distributed axisymmetric load acts on the surface of coating–substrate structure. Refer to the cylinder coordinate system($o-r\theta z$), where the origin o is located at the center of the interface between the coating and half-space. The shear modulus of coating varies as an arbitrary continuous function of $z, \mu(z)$. Because the substrate is much thicker than the coating, it is reasonable to treat it as an elastic half-space.

The new multi-layer model divides functionally graded coatings into N sub-layers as shown in Fig. 2. The shear modulus $\mu(z)$ in each sub-layer is assumed to vary as an exponential function form

$$\mu(z) \approx \mu_j(z) = a_j e^{b_j z}, \; h_j \le z \le h_{j-1}, \; j = 1, 2, ...N \tag{1}$$

$$\mu(h_j) = \mu_j(h_j), \tag{2}$$

$$a_j = \mu(h_j) e^{-\ln[\mu(h_{j+1})/\mu(h_j)] h_j/(h_{j+1}-h_j)}, \; b_j = \ln[\mu(h_{j+1})/\mu(h_j)]/(h_{j+1}-h_j) \tag{3a, b}$$

where h_j is the z coordinate at the end of layer j. Poisson's ratio in each sub-layer is assumed to be a constant ν_j.

In each sub-layer, the equilibrium equations are represented as [2]

$$(k_j+1)\left\{\frac{\partial^2 u_j}{\partial r^2} + \frac{1}{r}\frac{\partial u_j}{\partial r} - \frac{1}{r^2}u_j + \frac{\partial^2 w_j}{\partial r \partial z}\right\} + (k_j-1)b_j\left(\frac{\partial u_j}{\partial z} + \frac{\partial w_j}{\partial r}\right)$$
$$+(k_j-1)\left\{\frac{\partial^2 u_j}{\partial z^2} - \frac{\partial^2 w_j}{\partial r \partial z}\right\} = 0, \; j = 1, 2, ...N$$
$$(k_j+1)\left\{\frac{\partial^2 u_j}{\partial r \partial z} + \frac{1}{r}\frac{\partial u_j}{\partial z} + \frac{\partial^2 w_j}{\partial z^2}\right\} - (k_j-1)b_j\left(\frac{\partial^2 u_j}{\partial r \partial z} - \frac{\partial^2 w_j}{\partial r^2}\right)$$
$$-\frac{(k_j-1)}{r}\left\{\frac{\partial u_j}{\partial z} - \frac{\partial w_j}{\partial r}\right\} + (3-k)b_j\left(\frac{\partial u_j}{\partial r} + \frac{u_j}{r}\right) + (k_j+1)b_j\frac{\partial w_j}{\partial z} = 0,$$
$$j = 1, 2, ...N \tag{4a, b}$$

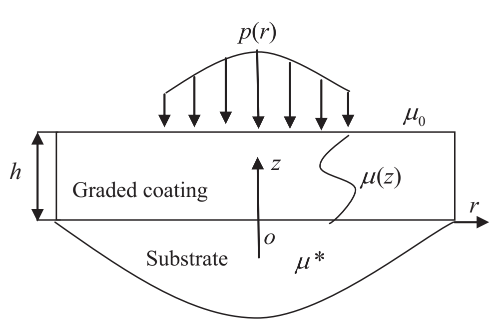

Fig. 1. The functionally graded coating–substrate structure under axisymmetric surface loading.

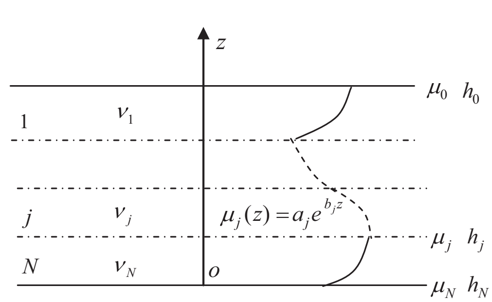

Fig. 2. The new multi-layer model for the graded coatings.

where u_j and w_j are the displacement components in the radial and z axial directions in layer j; r and z are the variables of the cylindrical coordinate system; $k_j = 3 - 4\nu_j$.

Applying the Hankel transform to Eq. (4) and defining $D = d/dz$, we can obtain the differential equation as follows:

$$\{(k_j-1)D^2 + b_j(k_j-1)D - (k_j+1)s^2\}\tilde{u}_j - \{2sD + b_j s(k_j-1)\}\tilde{w}_j = 0, \tag{5a}$$

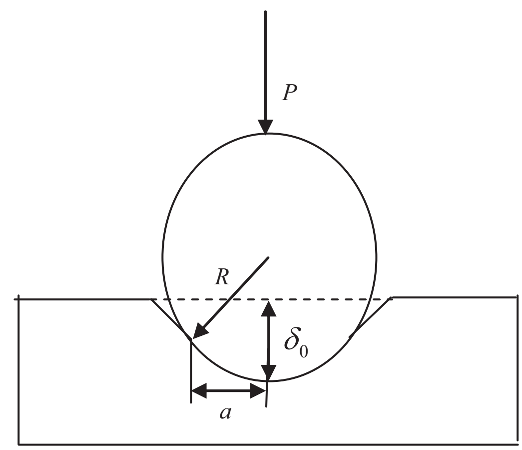

Fig. 3. The graded coatings indented by a rigid spherical punch.

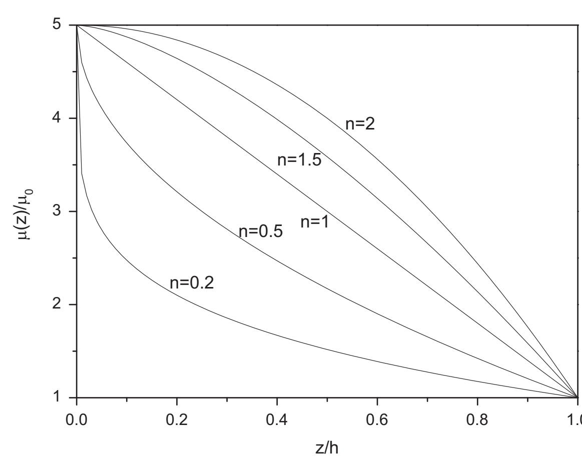

Fig. 4. The shear modulus variations follow a power law with $n=0.2$, 0.5, 1, 1.5 and 2 when $\mu_0/\mu* = 1/5$.

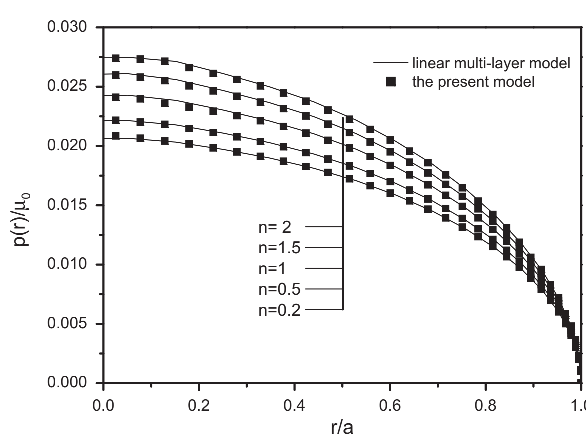

Fig. 5. The contact pressure on the surface of the FGMs coating is described for selected values of n using the present multi-layer ($N=12$) and the linear multi-layered model ($N=6$).