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Short Communication

Hysteresis friction and nonlinear viscoelasticity of rubber composites

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ABSTRACT

Evaluation of tire friction is essential from the perspective of enhancing vehicle performance and improving its safety. Earlier investigations have identified rubber adhesion and hysteresis as two fundamental mechanisms underlying tire friction. This contribution aims at analyzing the effect of nonlinear viscoelastic behavior of tire treads on their hysteresis friction coefficient. Tire tread is a composite material made of synthetic or natural rubber reinforced with particles such as carbon black or silica. Hysteresis that refers to energy dissipation during a cyclic deformation is an essential feature of mechanical behavior of rubber composites. The effect of nonlinear viscoelasticity of filled rubbers on the hysteresis friction of tire treads is not well understood. In this contribution, we use an analytical model to predict the hysteresis friction of filled rubbers sliding on a rough surface. The model accounts for the nonlinear viscoelasticity of the slider subjected to finite strains. It will be shown that the kinetic coefficient of friction directly correlates with the magnitude of strain. Comparison of model results against the experimental measurements for tire tread materials reveals that the predicted friction coefficients become comparable with experimental data at large strains (8–10%). This study suggests that the nonlinear viscoelasticity of tire treads needs to be taken into account in order to have a more realistic assessment of frictional performance of tires.

1. Introduction

Sliding friction of rubbers has two distinct components, adhesive and hysteresis friction [1–3]. While the adhesive component of friction is due to the adhesion between the exposed atoms on the sliding surfaces [4], the hysteresis friction is an attribute of the viscoelastic nature of the (filled) rubber. The adhesive friction is controlled by the finite lifetime of physical bonds between molecules at the rubber-substrate interface and thus, is assumed to be dominant at slow sliding velocities. At higher sliding velocities, the compliant tire treads undergo substantial deformations to envelope around and flow over the counter-surface asperities, at time scales comparable with their characteristic viscoelastic relaxation time. This leads to a hysteresis-like dissipation of energy during stochastic local excitation of rubber by the asperities. Theoretical foundations of hysteresis friction is developed in the seminal works of Persson [5] and Heinrich and Klüppel [6] (H-K model). In a nutshell, these theories consider a linear viscoelastic body dragged on a rough substrate with a constant force F_f. The friction is related to the energy dissipation during the sliding motion of the body, representing the tire tread, at constant velocity of v. Here, we only recall the key results of the H-K model in which the viscoelastic response of the sliding body is presented by a 1D linear element with a complex modulus of $E^*(\omega) = E'(\omega) + iE''(\omega)$. The frequency ω and time-dependent stress and strain in the element are connected via Fourier-transforms $\sigma(t) = (1/2\pi)\int \hat{\sigma}(\omega)e^{-i\omega t}d\omega$ and $\epsilon(t) = (1/2\pi)\int \hat{\epsilon}^*(\omega)e^{i\omega t}d\omega$. This way, the energy dissipated due to the sliding reads [6]

$$\Delta E_d = \frac{vt^*}{2(2\pi)^2}\int E''(\omega)\, S(\omega)\omega\, d\omega \tag{1}$$

where t^* denotes the time during which the asperity-induced stress acted upon the body and $S(\omega)$ is the power spectral density; i.e., the Fourier transform of the auto-correlation function between the strains induced by asperities. Assuming that the block is subjected to the apparent normal stress σ_0 in the contact zone and dissipation takes place within the average penetration depth $\langle z_p\rangle$ (Fig. 1(a)), the hysteresis friction coefficient can be expressed as [6]

$$\mu = \frac{\langle z_p\rangle}{2(2\pi)^2\sigma_0 v}\int E''(\omega)S(\omega)\omega\, d\omega \tag{2}$$

It is well known that the incorporation of silica or carbon black particles into a natural or synthetic rubber matrix strongly modifies its viscoelastic characteristics and leads to emergence of nonlinear responses such as the *Mullins* and *Payne* effects [7]. The latter refers to the

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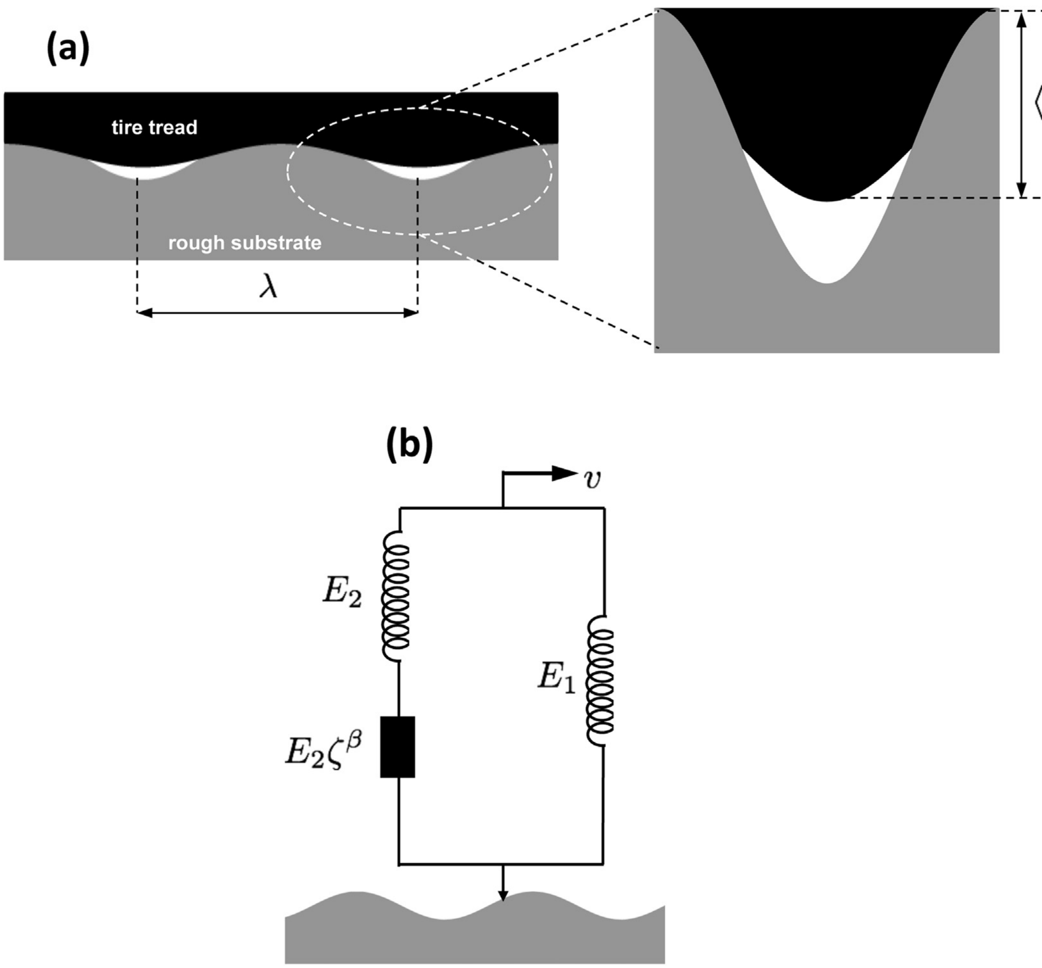

Fig. 1. (a) Rubber block sliding on a rough substrate with Westergaard profile. (b) 1D rheological model representing the nonlinear viscoelasticity of the filled rubber block [12].

strong nonlinearity in storage and loss modulus of the filled rubbers during oscillatory deformations with increasing the strain amplitude [8]. For example, in a typical strain-sweep test on filled styrene butadiene rubber, an increase in the amplitude of strain is followed by a decrease in the storage modulus. The loss modulus, on the other hand, initially increases with strain, reaches a maximum and then decreases with further increase in the strain amplitude [9]. The Payne effect has been the subject of extensive investigations on both experimental and theoretical aspects. Compounding of surface-active particles like silica with rubber is often followed by formation of particle aggregates or large agglomerates. Strain-induced nonlinearity is traced to the disruption of these disorderly grown structures [10]. Alternatively, the emergence of viscoelastic nonlinearity can be attributed to desorption of unstable bonds formed between rubber matrix and filler surface and promotion of polymer disentanglement from the particle surface at large deformations [11].

If the viscoelastic moduli of filled rubbers are strain-dependent, then it follows from Eq. (2) that the hysteresis friction would also depend on the local strain experienced by the sliding rubber. If strain is large, the nonlinear viscoelasticity must be taken into account for an accurate estimation of dissipated energy. While measurement of strain at the rubber-substrate interface is understandably a difficult task, an analytical model of rubber friction that accounts for viscoelastic nonlinearity may provide a better understanding of the state of strain. In this paper, we replace the 1D linear viscoelastic element in H-K model with a nonlinear model in order to understand how friction coefficient correlates with the local strain.

2. Model

Lion et al. [12] proposed a 1D model to link the variation of viscoelastic moduli to the amplitude of applied strain. They adopted an approach originally developed by Valanis [13] in which the physical time t is replaced by an intrinsic time scale $z(t, \omega, \epsilon)$, which depends on the frequency ω and strain amplitude ϵ. As a result, the constitutive equation in this model is expressed in terms of the fractional time derivatives of field quantities. Fig. 1(b) shows the 1D rheological model representing the viscoelasticity of sliding body. E_1 and E_2 are the spring constants controlling the equilibrium stress, whereas β and ζ characterize the nonlinear viscoelastic nature of the overstress in addition to the equilibrium stress. The constitutive equation for this rheological element can be expresses as [12]

$$\sigma(z) + \zeta^{\beta}\frac{d^{\beta}\sigma(z)}{dz^{\beta}} = (E_1 + E_2)\zeta^{\beta}\frac{d^{\beta}\epsilon(z)}{dz^{\beta}} + E_1\epsilon(z) \tag{3}$$

Assuming $z(t) = at$ with

$$a = 1 + \frac{2}{\pi}\, b\, \epsilon\, (\omega\tau)^{\alpha} \tag{4}$$

then the storage and loss moduli follow [12]

$$E'(\omega) = E_1 + \frac{E_2\left(\left(\frac{\omega\,\zeta}{a}\right)^{2\beta} + \left(\frac{\omega\,\zeta}{a}\right)^{\beta}\cos\left(\frac{\beta\,\pi}{2}\right)\right)}{1 + \left(\frac{\omega\,\zeta}{a}\right)^{2\beta} + 2\left(\frac{\omega\,\zeta}{a}\right)^{\beta}\cos\left(\frac{\beta\,\pi}{2}\right)} \tag{5a}$$