International Journal of Mechanical Sciences 80 (2014) 16–28

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Nonlinear vibration of imperfect eccentrically stiffened functionally graded double curved shallow shells resting on elastic foundation using the first order shear deformation theory

Dao Huy Bich, Nguyen Dinh Duc, Tran Quoc Quan*

Vietnam National University, Hanoi, Vietnam 144 Xuan Thuy, Cau Giay, Hanoi, Vietnam

ARTICLE INFO

Article history:
Received 1 October 2013
Received in revised form
16 November 2013
Accepted 19 December 2013
Available online 30 December 2013

Keywords:
Nonlinear dynamic response
Nonlinear vibration
FGM double curved shallow shells
The first order shear deformation theory
Stress function

ABSTRACT

This paper presents an analytical approach to investigate the nonlinear dynamic response and vibration of imperfect eccentrically stiffened FGM thick double curved shallow shells on elastic foundation using both the first order shear deformation theory and stress function with full motion equations (not using Volmir's assumptions). The FGM shells are assumed to rest on elastic foundation and subjected to mechanical and damping loads. Numerical results for dynamic response of the FGM shells are obtained by Runge–Kutta method. The results show the influences of geometrical parameters, the material properties, imperfections, the elastic foundations, eccentrically stiffeners and mechanical loads on the nonlinear dynamic response and nonlinear vibration of functionally graded double curved shallow shells. The numerical results in this paper are compared with results reported in other publications.

1. Introduction

Functionally Graded Materials (FGM) are homogeneous composite and microscopic scale materials with the mechanical and thermal properties varying smoothly and continuously from one surface to the other. The properties of FGM shells are assumed to vary through the thickness of the structures. Due to the high heat resistance, FGMs have many practical applications such as reactor vessels, aircrafts, space vehicles, defense industries and other engineering structures.

Therefore, in the recent years, many investigations have been carried out on the dynamic and vibration of FGM shells. Strozzi and Pellicano [1] investigated the nonlinear vibrations of functionally graded circular cylindrical shells by using the Sanders–Koiter theory. Sepiani et al. [2] studied the vibration and buckling analysis of two-layered functionally graded cylindrical shell, considering the effects of transverse shear and rotary inertia. Nonlinear buckling analysis of FGM shallow spherical shells under pressure loads was presented by Ganapathi [3] by using finite element method, geometric nonlinearity is assumed only on the meridional direction in strain–displacement relations. The nonlinear vibration of heated bimetallic shallow shells of revolution is presented in work [4] of Wang and Song. Zhao and Liew extended their previous works on isotropic conical panels to analyze the free vibration of functionally graded conical shells by using a meshless method [5]. Vibration analysis of ring-stiffened conical-cylindrical spherical shells based on a modified variational approach is investigated by Qu et al. [6]. Sheng and Wang [7] have considered the nonlinear vibration control of functionally graded laminated cylindrical shells based on Hamilton's principle, Von Karman nonlinear theory and constant-gain negative velocity feedback approach. Haddadpour et al. [8] obtained the free vibration analysis of functionally graded cylindrical shells including thermal effects. Loy et al. [9] also focused the vibration of functionally graded cylindrical shells. Xiang et al. [10] used Love's first approximation theory to analyze the natural frequencies of rotating functionally graded cylindrical shells. An analysis on the nonlinear dynamics of a clamped–clamped FGM circular cylindrical shell subjected to an external excitation and uniform temperature change is presented in [11]. Hong [12] investigated the functionally graded material shell with mounted magnetostrictive layer under thermal vibration by using the generalized differential quadrature (GDQ) method. Chandrashekhar et al. [13] focused on nonlinear vibration analysis of composite laminated and sandwich plates with random material properties. Huang and Shen [14] studied nonlinear free and forced vibration of simply supported shear deformable laminated plates with piezoelectric actuators. Huang and Han [15] presented nonlinear dynamic buckling problems of functionally graded cylindrical shell subjected to dependent axial load by using Budiansky–Roth dynamic buckling criterion [16]. Rafiee et al. [17] also published the results on the nonlinear vibration and dynamic response of simply

* Corresponding author.
E-mail address: quantq_530@vnu.edu.vn (T. Quoc Quan).

0020-7403/$ - see front matter Crown Copyright © 2014 Published by Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijmecsci.2013.12.009

supported piezoelectric functionally graded material shells under combined electrical, thermal, mechanical and aerodynamic loading. Sofiyev [18,19] used the large deformation theory with von Karman–Donnell-type of kinematic nonlinearity to study the nonlinear dynamics of FGM truncated conical shells. The vibration of FGM thin cylindrical shells with exponential volume law has been investigated by Shah et al. [20]. Amabili [21,22] studied the nonlinear vibrations of isotropic curved panels using the Donnell–Novozhilov shell theories. In these papers, the Lagrangian approach was utilized to obtain the equations of motion including geometric imperfections and the in-plane inertia effects. Nonlinear vibrations of functionally graded shallow shells under a concentrated force were studied by Alijani et al. [23]. Bich et al. [24] investigated the nonlinear vibration of functionally graded circular cylindrical shells based on improved Donnell equations. Based on the first-order shear deformation theory, Tornabene [25] studied the dynamic behavior of moderately thick functionally graded conical, cylindrical shells and annular plates. Additionally, Tornabene et al. [26,27] investigated vibration of FGM and laminated doubly curved shells and panels. Based on physical neutral surface and high order shear deformation theory, Zhang [28] analyzed the modeling and analysis of FGM rectangular plates. Patel et al. [29] considered the free vibration analysis of functionally graded elliptical cylindrical shells using higher-order theory. Up to date, a number of recent publications focused on the dynamic of eccentrically stiffened doubly curved FGM shallow shells using the stress function and Volmir's hypothesis but they all use classical shell theory. Bich et al. studied nonlinear postbuckling and dynamic of eccentrically stiffened functionally graded shallow shells, panel and circular cylindrical shells [30–33]. Duc et al. [34,36] investigated nonlinear postbuckling and nonlinear dynamic response of imperfect eccentrically stiffened doubly curved FGM shallow shells on elastic foundations.

When using higher order shear deformation theories to study the nonlinear dynamic analysis and vibrations of thick plates and shells, Volmir's hypothesis is useless. Chorfi and Houmat [37] investigated nonlinear free vibrations of FGM doubly curved shells with an elliptical plan-form using first order shear theory. Matsunaga et al. [38] investigated free vibrations and stability of FGM double curved shallow shells according to a 2-D higher order deformation theory. Noted that in mentioned above references [37,38], all authors used displacements functions. There is no any publication using simultaneous combination of the higher order shear deformation theory and stress function to investigate dynamic for eccentrically stiffened FGM double curved shallow shells. This is the first paper presenting an analytical approach to investigate the nonlinear dynamic response and nonlinear vibration of FGM shells using both the first order shear deformation theory, smeared stiffeners Lekhnitsky's technique and stress function. Numerical results for dynamic response of the FGM shells are obtained by fourth-order Runge–Kutta method.

2. Double curved FGM shallow shell on elastic foundation

Consider a FGM double curved thin shallow shell of radii of curvature R_x,R_y length of edges a, b and uniform thickness h. A coordinate system(x, y, z)is established in which (x, y) plane on the middle surface of the shell and z on thickness direction $(-h/2 \le z \le h/2)$ as shown in Fig. 1.

For the FGM shell, the volume fractions of constituents are assumed to vary through the thickness according to the following power law distribution (P-FGM):

$$V_c(z) = \left(\frac{2z+h}{2h}\right)^N; \quad V_m(z) = 1 - V_c(z), \tag{1}$$

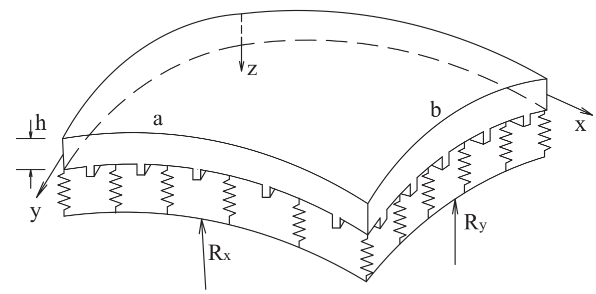

Fig. 1. Geometry and coordinate system of P-FGM double curved shallow shell on elastic foundation.

where N is volume fraction index $(0 \le N < \infty)$. Effective properties Pr_{eff} of FGM shell are determined by linear rule of mixture as

$$\mathrm{Pr}_{eff}(z) = \mathrm{Pr}_c V_c(z) + \mathrm{Pr}_m V_m(z), \tag{2}$$

in which Pr denotes a material property, and subscripts m and c stand for the metal and ceramic constituents, respectively. Specific expressions of elastic modulus E, Poisson ratio ν and density ρ are obtained by substituting Eq. (1) into Eq. (2) as

$$[E(z),\ \nu(z), \rho(z)] = [E_m,\ \nu_m, \rho_m] + [E_{cm}, \nu_{cm},\ \rho_{cm}]\left(\frac{2z+h}{2h}\right)^N, \tag{3}$$

where

$$E_{cm} = E_c - E_m, \nu_{cm} = \nu_c - \nu_m, \rho_{cm} = \rho_c - \rho_m. \tag{4}$$

It is evident from Eqs. (3) and (4) that the upper surface of the panel $(z = -h/2)$ is metal-rich, while the lower surface $(z = h/2)$ is ceramic-rich, and the percentage of metal constituent in the panel is enhanced when N increases.

Assume that the shell is reinforced by eccentrically longitudinal and transversal homogeneous stiffeners with the elastic modulus E_0. In order to provide the continuity between the shell and stiffeners, the full metal stiffeners are put at the metal-rich side of the shell thus $E_0 = E_m$ and conversely full ceramic ones at the ceramic-rich side, so that $E_0 = E_c$ [24,30–35].

The shell–foundation interaction is represented by Pasternak model as

$$q_e = k_1 w - k_2 \nabla^2 w, \tag{5}$$

in which $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$, w is the deflection of the panel, k_1 is Winkler foundation modulus and k_2 is the shear layer foundation stiffness of Pasternak model.

3. Theoretical formulation

In this study, the first-order shear deformation theory and the Lekhnitsky smeared stiffeners are used to establish governing equations and to determine the nonlinear vibration of FGM thick shallow double curved shells.

The strain–displacement relations taking into account the von Karman nonlinear terms are [39,40]

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{pmatrix} + z \begin{pmatrix} \chi_x \\ \chi_y \\ \chi_{xy} \end{pmatrix}, \quad \begin{pmatrix} \gamma_{xz} \\ \gamma_{yz} \end{pmatrix} = \begin{pmatrix} w_{,x} + \phi_x \\ w_{,y} + \phi_y \end{pmatrix}, \tag{6}$$

with

$$\begin{pmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{pmatrix} = \begin{pmatrix} u_{,x} - w/R_x + w_{,x}^2/2 \\ v_{,y} - w/R_y + w_{,y}^2/2 \\ u_{,y} + v_{,x} + w_{,x} w_{,y} \end{pmatrix}, \quad \begin{pmatrix} \chi_x \\ \chi_y \\ \chi_{xy} \end{pmatrix} = \begin{pmatrix} \phi_{x,x} \\ \phi_{y,y} \\ \phi_{x,y} + \phi_{y,x} \end{pmatrix}, \tag{7}$$