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Axial–torsional vibrations of rotating pretwisted thin walled composite beams

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ABSTRACT

Axial–torsional vibrations of rotating pretwisted thin-walled composite box beams exhibiting primary and secondary warping are investigated. Considering the nonlinear strain–displacement relations, the coupled nonlinear axial–torsional equations of motion are derived using Hamilton's principle. Ignoring the axial inertia term leads to differential equation of motion in terms of elastic torsion in the case of axially immovable beams. Centrifugal load in the presence of material anisotropy and pretwist angle leads to an induced static torque. The nonlinear equation should be linearized about the corresponding equilibrium state to obtain the linear differential equation of motion. Extended Galerkin's method is utilized to achieve the proper eigenvalue problem. The results obtained in this paper seek to clarify the individual and collective effects of axial loading, pretwist, stagger and fiber angles on the torsional behavior of the non-uniform thin-walled composite blades. The results are compared to available analytical and experimental results in the literature which reveals excellent agreements. The outcomes of this study are expected to offer better predictions of the dynamic behavior of this kind of structures in general, and in design of rotor blades of turbo-machinery, in particular.

1. Introduction

Torsional vibrations of beam-like structures have important role in advanced engineering structures. Predominantly torsional oscillations occur during stall induced vibrations and stall flutter of helicopter and wind turbine blades [1]. The ONERA model has been developed to compute the unsteady aerodynamic loads of a pitching airfoil in the dynamic stall regime [2]. Also, single degree of freedom torsional flutter has been observed in turbo-machinery blades as a result of their high mass ratio (structure to fluid) [3,4]. Tension–torsion coupling is the main reason of untwist phenomenon in rotating blades [5,6]. Furthermore, the concept of passive blade twist control was developed through the use of composite materials with proper tension–torsion elastic coupling, as implemented in the XV-15 tilt rotor aircraft. According to this technology, structural tailoring is utilized in the design of two different blade twist distributions corresponding to two rotor speeds (helicopter and airplane flight modes) to improve the aerodynamic performance [7,8]. Also, it should be noted that the tension–torsion is the key element in the torsional chatters of twist drills [9]. More recently, a new application exploiting the beam's inherent axial–torsional coupling has also been developed in the design of piezoelectric ultrasonic motors [10].

In order to ensure performance advantages, rotor blades are usually twisted. In the analysis of pretwisted blades, the collective influence of centrifugal axial force and pretwist angle on the torsional deformations should be considered. Buckley [11] and Wagner [12] pointed out to the increasing effect of axial load in the torsional rigidity of beam. If the beam is assumed to be consist of straight filaments in the untwisted state, then by twisting the beam these filaments will run through the cross section in a twisted pattern. The axial stress which acts along the filament will produce a couple in the twisted state about the neutral axis which is known as "Wagner effect". Based on this hypothesis, Houbolt and Brooks [13] pointed out to two linear terms in the governing equations of bending–bending–torsion of pretwisted blades related to the effects of axial load on the torsion of blades: a moment that stiffens the beam due to tensile stress coupled with elastic twist and a moment that untwist the bar under the collective influence of tensile load and pretwist angle. Ohtsuka [5] using a generalized beam theory including warping obtained theoretical and experimental results indicating untwist of blade under tensile stress. The paper by Washizu [14] represented a solid frame in the analysis of pretwisted and curved beam. At this situation, the research works by Rosen [15,16] also should be addressed. In the context of nonlinear elasticity, Hodges [6] utilized the tensorial relationships for transforming the quantities evaluated in the curvilinear coordinate

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system to the orthogonal local one. He concluded that the two components of untwist are of the same order of magnitude and the effect of torsion–tension coupling in the pretwisted bars is significant. Hodges and co-workers [17–19] have continued to study this effect which has been called trapeze effect. Fulton and Hodges [17] studied the nonlinear aeroelasticity of composite helicopter rotor blades in hover considering trapeze effect. Hodges et al. [18] presented a nonlinear sectional analysis of an anisotropic pretwisted strip. Variational asymptotic method has been used to reduce the laminated shell theory into a nonlinear one dimensional theory. Since the trapeze effect stems from the non-linearity of the cross-sectional analysis, Popescu and Hodges [19] represented a nonlinear numerical cross-sectional analysis based on the variational-asymptotic method which is capable to capture trapeze effect in cross sections of arbitrary geometry and generally anisotropic material. However, the majority of research works in this topic came into conclusion that trapeze effect should be considered in the analysis of pretwisted beams under axial loading. Interestingly at the same time the importance of nonlinear shortening term in the torsional vibrations of beam have been outlined (see Refs. [20–25]).

Modeling and dynamic behavior of axially loaded pretwisted beams have remained an important research topic during the last decade. Sakar and Sabuncu [26] investigated the dynamic stability of rotating pretwisted aerofoil cross-section blade. They studied the effects of various parameters such as shroud dimensions, pretwist and stagger angles, rotational speed and distance of shear center from the centroid on the stability of the blade. Vibration and parametric stability of spinning twisted beams under compressive axial pulsating loads have been investigated by Chen [27]. Sinha and Turner [28] implemented thin shell theory in the vibration analysis of typical turbo-machinery cantilevered airfoil blade subjected to a quasi-static axial load. Saravia et al. [29] linearized a geometrically nonlinear total Lagrangian finite element model with seven degrees of freedom per node to study the free vibration and dynamic stability of thin-walled rotating composite beams. This finite element formulation was based on a thin-walled beam theory that takes into account material anisotropy, shear deformation and warping inhibition. Dynamic behavior of rotating tapered carbon nanotube embedded polymer composite beams has been investigated by Deepak et al. [30] using spectral finite element formulation. Yao et al. [31] studied nonlinear dynamic responses of the pretwisted thin-walled rotating beam with varying rotating speed under high-temperature supersonic gas flow. The aerodynamic load has been modeled using first-order piston theory. The method of multiple scales is exploited to derive the four-dimensional averaged equation for the case of 1:1 internal resonance and primary resonance. Sari and Butcher [32] represented the vibration analysis of non-rotating and rotating Timoshenko beams with damaged boundaries using the Chebyshev collocation method.

Recently in the context of torsional vibrations of beam, Salim and Davalos [33] expanded the Vlasov's theory to perform the linear analysis of open and closed sections laminated composite beams considering warping–torsion terms. Mohri et al. [34] investigated a nonlinear model for large torsion analysis of Thin Walled Beam (TWB) including shortening effect, pre-buckling deflections and flexural–torsional couplings. This model is extended further to finite element formulation and then the corresponding nonlinear equations are solved using the incremental iterative Newton–Raphson method. Shin and Kim [35] obtained an exact solution for twist angle and fiber stresses of TWB made of composite materials with single and double-celled sections subjected to torsional moment. Liu et al. [36] studied the axial–torsional vibration of pretwisted beams and obtained a set of criteria for checking the validity of the simplifying assumptions used in the prismatic beam warping function. Sapountzakis and Tsipiras [37] obtained a boundary element solution for the torsional vibration problem of bars of arbitrary doubly symmetric cross-section. The influence of delamination on the trapeze effect in the case of anisotropic pretwisted beam is investigated by Prasad and Harursampath [38]. More recently, Sina et al. [39] analyzed the nonlinear normal modes of torsional vibrations of TWB and their stability using semi-analytic methods.

In spite of the tremendous research activity in this field, most studies were accomplished in the context of solid isotropic beam. Also, there are few studies that include the effects of anisotropy coupled to that of pretwist and stagger angles in the torsional analysis of rotating blades. To the best of the author's knowledge, the majority of numerical results concerning the torsional vibrations of pretwisted rotating beams are obtained by neglecting nonlinear terms in the governing equations (see Refs. [40–47]). As mentioned in the second paragraph, the collective effect of pretwist angle and centrifugal force leads to a static torque which tends to untwist the blade. This induced torque has another counterpart in the case of rotating composite blade due to extension–twist elastic coupling [45,46]. Hence, the blade may possess certain static torsion and the governing nonlinear equation of motion should be linearized around this static equilibrium state. Upon the results of this study, without considering this static equilibrium point the obtained results have both qualitative and quantitative errors. In summary the aim of this study is to

(i) represent the nonlinear equations governing axial–torsional vibrations of pretwisted rotating TWB in the presence of material anisotropy,
(ii) demonstrate the importance of induced static torque on the torsional vibrations of centrifugally stiffened composite TWB,
(iii) study the individual and collective effects of pretwist, stagger and fiber angles on the torsional vibrations of rotating composite TWB, and
(iv) represent some numerical results concerning the free torsional vibrations of rotating non-uniform TWBs.

The structural model considered here, incorporates a number of non-classical effects including primary and secondary warping, non-uniform torsional model and rotary inertia. The governing differential equations of motion for TWB featuring tension–torsion elastic coupling are derived using Hamilton's principle and the Galerkin's method is utilized to construct the mass and stiffness matrices.

2. The equations of motion

An initially twisted rotating thin walled box beam with the length of L, width of c, height of b, thickness of h, pretwist angle of β, hub radius of R_0 and constant angular velocity of Ω is considered as shown in Fig. 1. The model is limited to twisted bars with cross sections having two-fold symmetry. Otherwise the torsion and axial extension are coupled with bending, which is not included in the present analysis. The pretwist angle assumed to varied linearly along the blade span ($\beta=\beta_0(z/L)$). Reference coordinate defined as (x^p, y^p, z) as a local coordinate associated with the beam and another coordinate (s, n, z), is used to define complex cross section profiles (Fig. 2). The z-axis is located as to coincide with the locus of symmetrical point of the box beam's cross-section along the beam span.

It should be noted that in the kinematical relations, only the expressions related to the elastic torsion will be retained. The position of the point after deformation, subscript d, can be