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Isotropic damage analysis of frictional contact problems using quadratic meshless boundary element method

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ABSTRACT

In this paper, a quadratic meshless boundary element formulation for isotropic damage analysis of contact problems with friction is presented. To evaluate domain-related integrals due to the damage effects, the radial integration method (RIM) based on the use of the approximating the normalized displacements in the domain integrals by a series of prescribed radial basis functions (RBF) is employed. An exponential evolution equation for the damage variable is adopted. The details of coupling the different systems of equations for each body in contact under the several contact conditions are given to obtain the overall system of equations. Numerical examples covering shrink-fit and frictional punch problems are given to demonstrate the efficiency of the present meshless BEM.

1. Introduction

It is known that the boundary element (BE) method is a well-suited computational tool for linear elastic problems. Owing to its high resolution of stresses on the surface, the BE approach has been shown to be accurate in problems involving stress concentration, fracture mechanics and contact analysis. However, its extension to non-homogeneous and nonlinear damage problems is not a straightforward task, since it gives rise to additional domain-related integrals.

In this work, a quadratic meshless boundary element formulation for isotropic damage analysis of contact problems with friction is given. To transform domain integrals into boundary integrals, the radial integration method (RIM), developed by Gao [1], based on the use of approximating the normalized displacements in the domain integrals by a combination of radial basis functions and polynomials in terms of global coordinates, leading to a meshless scheme, is employed. An exponential evaluation equation for the damage variable, which is the ratio of the damaged area to the total area of the material considered, is adopted. The different systems of equations for each body in contact are united under the contact conditions, including shrink-fit applications, infinite friction, frictionless and Coulomb friction. The shear modulus is a function of the stresses; therefore, two iterative schemes are combined in such a way that contact iterations are carried out for each damage iteration step. Numerical examples covering shrink-fit and frictional cases are given to demonstrate the efficiency of the present meshless BEM.

2. BE analytical formulations

For isotropic, homogenous and linear elastic solids with the shear modulus μ being dependent on the damage variable D with constant Poisson's ratio ν, the scalar damage variable D, which is defined as the ratio of the damaged area to the total area of the considered body, is assumed to be a function of stress invariants [2]. The following damage evolution equation is adopted:

$$D = 1 - e^{-(\sigma_{eq}/\sigma_0)^m} \tag{1}$$

where m and σ_0 are material constants, and the equivalent stresses σ_{eq} in terms of deviatoric stresses S_{ij} is given by

$$\sigma_{eq} = \sqrt{\frac{3}{2} S_{ij} S_{ij}}, \quad S_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij} \tag{2}$$

The shear modulus can be then expressed as [2]

$$\mu(\sigma) = \mu_0(1-D) = \mu_0 e^{-(\sigma_{eq}/\sigma_0)^m} \tag{3}$$

where μ_0 is the shear modulus of the undamaged materials.

Without considering body forces, the displacement and stress boundary-integral equations for varying shear modulus are, respectively, given as [2–4]

$$C_{ij}(P)\tilde{u}_i(P) = -\int_S T_{ij}(P,Q)\tilde{u}_j(Q)dS_Q + \int_S U_{ij}(P,Q)t_j(Q)dS_Q + \int_\Omega (V_{ijk}(P,q))\tilde{u}_j(Q)d\Omega(q) \tag{4}$$

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$$\sigma_{ij}(p) = -\int_S T_{ijk}(p,Q)\tilde{u}_k(Q)dS_Q + \int_S U_{ijk}(p,Q)t_k(Q)dS_Q + \int_\Omega (V_{ijk}(p,q))\tilde{u}_k(Q)d\Omega(q) + F_{ijk}(p)\tilde{u}_k(p) \quad (5)$$

In these expressions U_{ij} and T_{ij} are the second-order displacement and traction tensors in the i direction at the field point Q or q due to an orthogonal unit load at the variable point P or p in the j direction. $\tilde{u}_i$ and t_i are displacement and traction, respectively. Capital letters are used to indicate that the point concerned lies on the boundary S. C_{ij} is the free-term tensor, whose components depend on the geometry, and Ω represents the solution domain. The kernel functions V_{ij} and V_{ijk} are given as [3]

$$V_{ij} = \frac{-1}{4\pi(1-\nu)r}\left\{\frac{\partial\tilde{\mu}}{\partial x_k}\frac{\partial r}{\partial x_k}\left[(1-2\nu)\delta_{ij} + 2\frac{\partial r}{\partial x_i}\frac{\partial r}{\partial x_j}\right] + (1-2\nu)\left(\frac{\partial\tilde{\mu}}{\partial x_i}\frac{\partial r}{\partial x_j} - \frac{\partial\tilde{\mu}}{\partial x_j}\frac{\partial r}{\partial x_i}\right)\right\} \quad (6)$$

$$V_{ijk} = \frac{-1}{4\pi(1-\nu)r^2}\left\{2\frac{\partial\tilde{\mu}}{\partial x_m}\frac{\partial r}{\partial x_m}\left[(1-2\nu)\delta_{ij}\frac{\partial r}{\partial x_k} + \nu\left(\delta_{ik}\frac{\partial r}{\partial x_j} + \delta_{jk}\frac{\partial r}{\partial x_i}\right) - 4\frac{\partial r}{\partial x_i}\frac{\partial r}{\partial x_j}\frac{\partial r}{\partial x_k}\right] + 2\nu\left(\frac{\partial\tilde{\mu}}{\partial x_i}\frac{\partial r}{\partial x_j} + \frac{\partial\tilde{\mu}}{\partial x_j}\frac{\partial r}{\partial x_i}\right)\frac{\partial r}{\partial x_k} - (1-4\upsilon)\frac{\partial\tilde{\mu}}{\partial x_k}\delta_{ij} + (1-2\nu)\left(2\frac{\partial\tilde{\mu}}{\partial x_k}\frac{\partial r}{\partial x_i}\frac{\partial r}{\partial x_j} + \frac{\partial\tilde{\mu}}{\partial x_j}\delta_{ik} + \frac{\partial\tilde{\mu}}{\partial x_i}\delta_{jk}\right)\right\} \quad (7)$$

The integral free-term F_{ijk} depends on the load point and it is given as

$$F_{ijk} = \frac{-1}{4\pi(1-\upsilon)}\left\{\delta_{ij}\frac{\partial\tilde{\mu}}{\partial x_k} + \delta_{ik}\frac{\partial\tilde{\mu}}{\partial x_j} + \delta_{jk}\frac{\partial\tilde{\mu}}{\partial x_i}\right\} \quad (8)$$

$\tilde{u}_i$ and $\tilde{\mu}$ are normalized displacements and shear modulus, respectively, and given as [2]

$$\tilde{u}_i(x) = \mu(x)u_i(x), \quad \tilde{\mu}(x) = \log\mu(x) \quad (9)$$

3. Numerical implementation

By discretizing the boundary S into a series of isoparametric quadratic elements, and introducing internal nodes and collocating the load point P at every boundary, a system of equation for boundary can be obtained. The evaluation of the boundary-only-related integrals is well covered in the literature and will be summarized here. The geometry can be described in terms of quadratic shape functions in a local co-ordinate axes system as follows (see for example Refs. [5,6]):

$$x_i(\xi) = \sum_{c=1}^{3} N_c(\xi)(x_i)_c \quad (10)$$

where N_c is the quadratic shape function and ξ is the local co-ordinate. Similarly, the displacement and traction vectors can be expressed in terms of quadratic shape functions. For two-dimensional formulation, the kernel functions contain singularities of the order of $1/r$ where r is the distance between the load point p and the field point Q. Therefore, the integrals become singular when p coincides with Q. It is important to devise accurate numerical integration schemes to evaluate the integrals in such cases, as it has a direct influence on the accuracy of the solutions. When p and Q do not coincide, the standard Gaussian quadrature formulae can be used, even if p and Q are in the same element. However, when p coincides with Q, the concept of rigid body motion can be employed (see for example Refs. [5,6]). To compute stresses at boundaries, the traction-recovery method is applied to Eq. (5).

For the evaluation of the domain-related integrals without resorting to the discretization of the domain into internal cells, the radial integration approach is adopted to transform them into boundary integrals. To do so, the normalized displacements are approximated by a series of prescribed basis functions used in the dual reciprocity method DRM [7], given as

$$\tilde{u}_i = \sum_A \alpha_i^A \phi^A(R/S_A) + a_i^k x_k + a_i^0 \quad (11)$$

where $R = ||x - x^A||$ is the distance from the application point x^A to the field point x, whereas S_A is the size of the support region for the radial basis function (RBF) at points x^A and α_i^A and α_i^k are coefficients to be determined. By considering computational experiences [3], the following 4th-order spline RBF is employed in the present formulation:

$$\phi^A(R/S_A) = \begin{cases} 1 \quad -6\left(\frac{R}{S_A}\right)^2 \quad +8\left(\frac{R}{S_A}\right)^3 \quad -3\left(\frac{R}{S_A}\right)^4 & 0 \le R \le S_A \\ 0 & R \ge S_A \end{cases} \quad (12)$$

In this work, according to computational experiences [3], the support size S_A is defined by selecting six application points x^A along each direction. After solving the resulting set of algebraic equations, the coefficients α_i^A and a_i^k appearing in Eq. (11) are obtained. By substituting the normalized displacements given in Eq. (11) into the domain integral for displacement in Eq. (4) and then applying the RIM, the domain radial integral for displacements is obtained. Similarly, the domain radial integral for stresses can be obtained. The details of the transformation of the domain integrals into the boundary integrals can be found in [2–4]. The radial integrals for the displacements are regular and they can be integrated numerically using the standard Gaussian quadrature integration procedure. However, the radial integrals for stresses are strongly singular when the load point approaches the field point. To evaluate these radial integrals, the technique, known as singularity separation technique [6], is employed.

It should be noted that the integration process is performed separately for each domain in contact. By taking each point on the boundary in turn as the load point and performing necessary numerical integrations, a set of linear algebraic equations is obtained for each contacting body boundary nodes and can be formed as follows:

$$[A][\tilde{u}] = [B][t] + [V][\tilde{u}] \quad (13)$$

The matrices $[A]$, $[B]$ and $[V]$ contain the integrals of the displacement, traction and domain kernel integrals, respectively. All matrices are fully populated. To arrive at the solution over boundary nodes for a single solution domain, Eq. (13) becomes

$$[\tilde{A}][\tilde{u}] = [B][t] \quad (14)$$

The matrix $[\tilde{A}]$ is a combination of $[A]$ and $[V]$. When internal points appear in the BE mesh. The matrix $[V]$ can be split into two matrices as: $[V]^b$, including boundary-only-related terms and $[V]^i$, internal-only-related terms. The following expression appears to be:

$$[A][\tilde{u}] = [B][t] + [V]^b[\tilde{u}]^b + [V]^i[\tilde{u}]^i \quad (15)$$

With further manipulation as in Eq. (13), this expression becomes

$$[\tilde{A}][\tilde{u}] = [B][t] + [V]^i[\tilde{u}]^i \quad (16)$$

To perform the necessary iterations, this expression requires only boundary and contact conditions, since the values of displacements at internal points are computed from the previous iteration step. It should be noted that matrices $[A]$ and $[B]$ appearing in the above-given expressions are functions of geometry and material properties, respectively. Thus they do not need to be re-calculated for each iteration.

4. BE contact formulation

In order to couple the different systems of equations obtained from the discretized BE equation for each body in contact, the contact conditions have to be imposed on the contacting node