International Journal of Non-Linear Mechanics 45 (2010) 463–473

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Transient MHD stagnation flow of a non-Newtonian fluid due to impulsive motion from rest

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ARTICLE INFO

Article history:
Received 9 January 2009
Received in revised form
5 October 2009
Accepted 1 January 2010

Keywords:
Unsteady
Non-Newtonian fluid
Power-law index
MHD
Analytical and numerical solutions

ABSTRACT

The transient boundary layer flow and heat transfer of a viscous incompressible electrically conducting non-Newtonian power-law fluid in a stagnation region of a two-dimensional body in the presence of an applied magnetic field have been studied when the motion is induced impulsively from rest. The non-linear partial differential equations governing the flow and heat transfer have been solved by the homotopy analysis method and by an implicit finite-difference scheme. For some cases, analytical or approximate solutions have also been obtained. The special interest are the effects of the power-law index, magnetic parameter and the generalized Prandtl number on the surface shear stress and heat transfer rate. In all cases, there is a smooth transition from the transient state to steady state. The shear stress and heat transfer rate at the surface are found to be significantly influenced by the power-law index N except for large time and they show opposite behaviour for steady and unsteady flows. The magnetic field strongly affects the surface shear stress, but its effect on the surface heat transfer rate is comparatively weak except for large time. On the other hand, the generalized Prandtl number exerts strong influence on the surface heat transfer. The skin friction coefficient and the Nusselt number decrease rapidly in a small interval $0 < t^* < 1$ and reach the steady-state values for $t^* \geq 4$.

1. Introduction

In recent years, the non-Newtonian fluids with or without magnetic field find an increasing applications in industries such as the flow of nuclear fuel slurries, liquid metal and alloys, plasma and mercury, lubrication with heavy oils and greases, coating of papers, polymer extrusion, continuous stretching of plastic films and artificial fibres and many others. During the past three decades there have been extensive research works on various aspects of non-Newtonian power-law fluids over bodies of different shapes which are documented in books by Skelland [1], Bird et al. [2] and Tanner [3]. Irvine and Karni [4] have presented an excellent review of non-Newtonian fluids. The steady viscous incompressible flow of a non-Newtonian power-law fluid on a two-dimensional body in the presence of a magnetic field was studied by Sarpkaya [5] and Djukic [6,7]. Andersson et al. [8] have considered the steady MHD flow of a power-law fluid over a linearly stretching surface. The flow and heat transfer of a power-law fluid over a uniform moving surface with a constant parallel free stream in the presence of a magnetic field have been studied by Kumari and Nath [9]. Liao [10] has obtained an analytical solution of the MHD of a non-Newtonian power-law fluid over a linearly stretching surface. Abel et al. [11] have considered the heat and mass transfer aspect of this problem. Abo-Eldahab and Salem [12] have examined the Hall effect on the MHD free convection flow of a non-Newtonian power-law fluid on a stretching surface. Recently, Zhang and Wang [13,14] have presented a mathematical analysis for the existence and uniqueness of the self-similar solution for two-dimensional MHD boundary layer flow of dilatant fluids ($N > 1$). However, the above problems deal with steady flows. Recently, Xu and Liao [15] have obtained the solution of the unsteady MHD viscous flow of a non-Newtonian power-law fluid caused by an impulsively stretching surface by using the homotopy analysis method (HAM). Also, Xu et al. [16] have used homotopy analysis method (HAM) to study the unsteady MHD flow of a viscous incompressible non-Newtonian power-law fluid near the forward stagnation-point region of a two-dimensional body. More recently, Kumari et al. [17] have investigated the unsteady MHD flow and heat transfer of a viscous incompressible electrically conducting non-Newtonian power-law fluid in the stagnation region of a two-dimensional body. Two situations were considered: (a) the flow is initially steady, and at $t > 0$ there is a step-change in the velocity of the potential flow, and (b) the velocity in the

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0020-7462/$ - see front matter © 2010 Published by Elsevier Ltd.
doi:10.1016/j.ijnonlinmec.2010.01.002

potential flow is time-dependent. The governing differential equations were solved numerically. It is evident from the literature survey that only a few analyzes are available for the transient flow and heat transfer problems of a non-Newtonian power-law fluid over a surface. To this end, it is worth mentioning that Garg and Rajagopal [18] obtained a pseudo-similarity solution for the flow of an incompressible fluid of second grade past a wedge with non-permeable walls, while Garg [19] reported such solutions for a permeable wedge with suction, respectively by augmenting the boundary conditions. It was shown in these papers that the advantage of augmenting the boundary conditions over the perturbation approach for small values of the dimensionless normal stress perturbation parameter ε is that the analysis is valid even for large values of ε, as shown by Garg and Rajagopal [20].

Since many flow and heat transfer problems of practical interest are unsteady either due to the impulsive change in the free stream velocity or surface velocity and (or) sudden change in wall temperature (or heat flux) or due to the time dependent variation in them, it is essential to know how the surface shear stress and heat transfer rate are affected by the nature of the non-Newtonian fluids characterized by the parameter N, magnetic field and Prandtl number in the entire time interval. Therefore, a parametric study showing the influence of these parameters on velocity and temperature as well as on the surface shear stress and heat transfer rate will be useful to chemical engineers involved with non-Newtonian flow problems.

This paper considers the transient flow and heat transfer of a viscous incompressible electrically conducting non-Newtonian power-law fluid in the stagnation region of a two-dimensional body with an applied magnetic field. We have studied the situation where prior to the time t=0, the body and the fluid are at rest and the wall and the fluid have same temperature T_∞. Then at t=0, the external stream is set into impulsive motion from rest and the body temperature is suddenly raised to $T_w(T_w > T_\infty)$. The partial differential equations governing the flow and heat transfer have been solved by both homotopy analysis method and finite-difference scheme. The computation has been carried out from the initial transient flow to the steady state flow. For some particular cases, analytical or approximate solutions have been obtained. The results have been compared with those of Xu et al. [16], Kumari et al. [17], Sparrow et al. [21], Pop [22], Nazar et al. [23] and Chen and Radulovic [24]. It may be remarked that our analysis supplements the results of the flow problem studied by Xu et al. [16] who presented the results for Newtonian and dilatant fluids ($N \geq 1$), whereas we have given the flow and heat transfer results mostly for pseudoplastic fluid ($N < 1$). The results presented here may be useful to chemical engineers in selecting appropriate non-Newtonian fluid as a working fluid so that the surface shear stress and heat transfer rate can be controlled. At present our results may not have any direct industrial application. However, they could be useful if the quantitative design procedures for industrial processing operations using the results of fluid-mechanical, rheological and molecular researches are developed. Although exact modelling of a physical situation, in general, is quite difficult, some simple mathematical model like the present one can express its average behaviour for some physical situations.

2. Non-Newtonian models

In this section, we have briefly discussed some of the non-Newtonian fluid models. Non-linear fluid rheology is encountered in several practical situations and the study of non-Newtonian fluid motion is an important topic. Among the most popular rheological models for non-Newtonian fluids is the power-law or Ostwald-de Waele model [25]. This model is a simple non-linear equation of state for inelastic fluids which includes linear Newtonian fluids as a special case. The power-law model provides an adequate representation of many non-Newtonian fluids over the most important range of shear rates. This, together with its apparent simplicity, has made it a very attractive model both in analytical and numerical research. The constitutive equation for a power-law fluid can be expressed as [25,26]

$$\boldsymbol{T} = -p\boldsymbol{I} + K(tr\boldsymbol{A}^2)^{\frac{n-1}{2}}\boldsymbol{A}. \tag{1}$$

Here, the Cauchy stress tensor $\boldsymbol{T}$ is expressed in terms of the pressure p, the material constant K, power-law index n and the identity matrix $\boldsymbol{I}$, while the first Rivlin–Ericksen tensor $\boldsymbol{A}$ is defined in terms of the velocity vector $\boldsymbol{V}$ as

$$\boldsymbol{A} = (grad\,\boldsymbol{V}) + (grad\,\boldsymbol{V})^T. \tag{2}$$

The above constitutive equation represents shear-thinning (pseudo-plastic) fluids for $n < 1$ and shear-thickening (dilatant) fluids for $n > 1$, whereas n=1 corresponds to Newtonian (i.e., linear) rheology. Pseudoplastic fluids such as water-based polymer muds and soap solutions and suspensions are non-Newtonian and can be represented by power-law model. However, this model is not suitable for elastic fluids.

In some industrial processes slightly viscoelastic fluid or highly viscoelastic fluid such as polymer melts, like high-viscosity silicone oils are used. The constitutive equation for viscoelastic homogenous fluid of second-order is given by Rivlin and Ericksen [27]

$$\boldsymbol{T} = -p\boldsymbol{I} + \mu\boldsymbol{A}_1 + \alpha_1\boldsymbol{A}_2 + \alpha_2\boldsymbol{A}_1^2, \tag{3}$$

where $\boldsymbol{T}$ is the stress tensor, p is the pressure, $\boldsymbol{I}$ is the identity matrix, α_1 and α_2 are the normal stress moduli and $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ are defined as

$$\boldsymbol{A}_1 = (grad\,\boldsymbol{V}) + (grad\,\boldsymbol{V})^T,$$

$$\boldsymbol{A}_2 = (d/dt)\boldsymbol{A}_1 + \boldsymbol{A}_1 \cdot grad\,\boldsymbol{V} + (grad\,\boldsymbol{V})^T \cdot \boldsymbol{A}_1. \tag{4}$$

Here $\boldsymbol{V}$ denotes the velocity field and d/dt is the material time derivative. Some assumptions on the sign of α_1 in Eq. (3) is required. For thermodynamic reasons, the material parameter α_1 must be positive [28]. If the fluid of second order modelled by Eq. (3) is to be compatible with thermodynamics and is to satisfy the Clausius–Duhem inequality for all motions and the assumption that the specific Helmholtz free energy of the fluid is a minimum when it is locally at rest, then

$$\mu \geq 0, \alpha_1 \geq 0, \alpha_1 + \alpha_2 = 0. \tag{5}$$

Rajagopal [29] has observed that for the viscoelastic fluids of second order, the equations of motion are, in general, one order higher than the Navier–Stokes equations and, in general, need additional boundary conditions to determine the solution completely. These issues were discussed in detail by Rajagopal [29,30], Rajagopal and Gupta [31] and Ariel [32]. It may be remarked that the use of the second-grade model governed by (3) is questionable since this simple rheological model is good only for slow flows with small levels of elasticity. But in many cases, the elasticity (or Weissenberg number) can be quite large [2]. Moreover, as mentioned in [28,33] there are some serious concerns about the sign and magnitude of model parameters appearing in a second-grade model such that the relevance of results obtained using this model is suspected even at small elasticity numbers.

In view of the limitations of the second-grade model mentioned above, it would be appropriate to use more realistic models such as upper-convected Maxwell, Phan-Thien-Tanner and Giesukus models [34] to simulate fluid flows. Recently, Sadeghy et al. [35,36] have studied Sakiadis and stagnation flows,