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The relationship between the force and separation of miniature magnets used in dentistry

Brian W. Darvell [a,1], *Brian H. Gilding* [b,*]

[a] *Department of Bioclinical Sciences, College of Dentistry, Kuwait University, Kuwait*
[b] *Department of Mathematics, College of Science, Kuwait University, Kuwait*

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ABSTRACT

Objective. Miniature magnets are used in dentistry, principally for the retention of prosthetic devices. The relationship between force and separation of a magnet and its keeper, or, equivalently, two such magnets, has been neither defined theoretically nor described practically in any detail suitable for these applications. The present paper addresses this lacuna.
Methods. A magnet is considered as a conglomeration of magnetic poles distributed over a surface or a solid in three-dimensional space, with the interaction of poles governed by the Coulomb law. This leads to a suite of mathematical models. These models are analysed for their description of the relationship between the force and the separation of two magnets.
Results. It is shown that at a large distance of separation, an inverse power law must apply. The power is necessarily integer and at least two. All possibilities are exhausted. Complementarily, under reasonable assumptions, it is shown that at a small distance of separation, the force remains finite.
Significance. The outcome is in accordance with practical experience, and at odds with the use of simple conceptual models. Consequences relevant to the usage of magnets in dentistry are discussed.

1. Introduction

Miniature magnets are used in dentistry for the retention of dentures and overdentures. In clinical orthodontics they are applied to treat misaligned teeth, stabilize the spacing between teeth, extrude impacted teeth, and correct open bite.

The complete loss of teeth has long been a problem, especially for the aged. The standard treatment, primarily to restore functionality, has been the provision of artificial dentures. Such dentures are intended to have a close fit to the soft tissue of the remaining dental arches in order to facilitate their retention by the force of suction arising from the narrowness of the intervening spaces and the viscosity of saliva [10]. Unfortunately, no matter how well prepared, that fit is soon inevitably lost. In the absence of the usual forces of mastication through the roots of the natural teeth onto the bone, that bone remodels and resorbs, and the soft tissue retreats accordingly, resulting in the dentures becoming mobile. Nevertheless, most recipients learn progressively to manipulate their dentures through the use of a combination of peri-oral musculature and tongue action, employing them effectively as

* *Corresponding author at*: Department of Mathematics, College of Science, Kuwait University, P.O. Box 5969, Safat 13060, Kuwait.
E-mail addresses: b.w.darvell@hku.hk (B.W. Darvell), gilding@sci.kuniv.edu.kw (B.H. Gilding).
[1] Present address: School of Dentistry, University of Birmingham, United Kingdom.
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tools for eating. Some, indeed, manage to do so when essentially there is no fit at all. Even so, loose dentures may still be a source of embarrassment and loss of quality of life. Many attempts and approaches have been used to remedy this situation, most recently through the use of endosseous implants, to which denture-bearing superstructures may be attached. Such treatments, while effective, are both invasive and expensive.

Often, the loss of teeth may be incomplete. Some remaining roots — attached as they are biologically and naturally, through the periodontal ligaments to the underlying bone — may be used as anchorages for dentures. Since the masticatory forces are again transmitted to the bone, resorption is reduced (locally at least). Again, there are a number of approaches that may be used, some of which are rigid fixations. These may have problems in terms of oral hygiene. More temporary fixation is therefore desirable, ensuring that the denture may be removed for cleaning. Of the latter type is that of the use of magnetic forces of attraction.

In essence, a miniature magnet is embedded in the denture base in the surface that will be in contact with the soft tissue, accurately to align with a 'keeper' of a magnetically-permeable alloy inserted into the remaining root, roughly even with the surface of the surrounding gum. Ideally, there are several such retentive devices. Thus, when the denture is fitted, the magnets match up with their keepers, and the prosthesis is held in place stably and securely, yet it may still be removed easily enough by hand. This approach really became viable with the advent of cobalt–samarium alloys. Latterly, the much stronger neodymium–iron–boron alloys are exclusively used in dentistry. Design is complicated by the fact that magnet alloys corrode readily in the mouth, such that cladding in a (non-magnetic) corrosion-resistant alloy or some other mechanically-resistant coating is essential. This puts a limit on the proximity between magnet and keeper that may be attained [1,18–22,40,41].

Such strong magnets have also found application in orthodontics, i.e. the application of controlled forces to teeth in order to correct their alignment. These forces must be sufficient to exceed the biological threshold for pressure-induced remodelling of the bone of the socket, yet not be so great as to cause damage. Ordinarily, these forces are applied through the agency of metal wire springs, although polymeric elastic devices may be used. Magnets are an alternative to generate the forces required, both in repulsion and attraction. These are bonded to the teeth, and can be used in combination with an arch-wire better to control movement. In cases where the spacing between incisors has been closed by other methods, the bonding of magnets to the anterior surface of the incisors is an effective means of stabilizing the closure. A further use is in the extrusion of impacted teeth. A small magnet is bonded to the unerupted tooth and another is embodied in a prosthesis. The attractive force between the two magnets then pulls the unerupted tooth through the soft tissue, and keeps the tooth in position for a period of retention. This has the advantage over mechanical methods that, once the tissues have healed after bonding of the magnet, there is no open wound [7,8,31,34,40–43].

Very early in the course of this development and clinical application, it was recognized that magnetic force decreases rapidly as the devices involved become separated. There have been various attempts reported to describe the relationship between the force acting and the separation between two magnets or between a magnet and keeper in both the prosthetic dentistry and orthodontic literature. With no suggestion of the physical basis for a plain assumption, it has been said that the force decreases first as the square and then as the cube of the distance of the air gap [4], reduces exponentially at a rate that is greater than linear but less than the square of the distance [3], follows the Coulomb law and is inversely proportional to the square of the distance [6,28,34,37,43–45], and, decreases *approximately* as the reciprocal square of the separation distance [9]. There seems to have been no subsequent correction or refinement of any of these.

On the basis of experimental work using diverse magnetic systems it has been reported that the force of repulsion of two magnets with opposed poles diminishes by the square of distance between them [16], that the breakaway force of a magnet and keeper and the air gap are inversely proportional [23], that the force of separated magnets increased inversely to the second power of the distance as a consequence of the force–distance relationship being hyperbolic [46], that for the force generated between magnets at separations larger than 2 mm, the classical Coulomb law of magnetic force was followed, and at 0 to 2 mm separation, an approximately inverse square root relationship was followed [15], that the force–distance curve of repelling magnets is hyperbolic, which is characteristic of Coulumb's law (*sic*) [5], that the force–distance relationship of commercially-available magnets and a magnetizable stainless steel plate approaches an inverse fourth power law at large separation, while at small separation the inverse square law applies [13], and that for separations of diverse magnets over 0.1 mm all analysed results show force–distance variation falling between $1/d^2$ and $1/d^4$, where d is the distance of separation, while for separations less than 0.1 mm the force varying as $1/d^2$ provides a close fit [39]. In the case of long thin dipoles, distinct from the real magnets considered previously, it has been found that the force is inversely proportional to $(d^c + h)^2$, where d is the separation of the magnet face and the keeper, h is an offset, and $c \approx 0.75$ [12]. Instances in which experimentally-derived curves have been presented without comment on their possible representation by a suitable mathematical expression include [2,22,24–27,29,32,33,35,36,44].

That, with key exceptions, there is an absence of a systematic study of the force–distance relationship of miniature magnets used in dentistry with any reference to actual physics is remarkable. Apart from recent investigations using the finite element method [24,35,39], attempts at physically-based mathematical modelling are limited to [11–13]. Prior to [11], the lack of an explanation for a finite breakaway force, i.e. that encountered when there is no separation, which is incompatible with any inverse proportionality law, is particularly noticeable.

In [13], modelling of a bar magnet as a dipole led to the conclusion that the force–distance relationship approaches an inverse fourth power law at large separation, while at small separation the inverse square law applies. It is questionable whether a real magnet may be viewed as a dipole. In [11,12] a magnet was modelled as a conglomeration of poles on a disc, the force being determined numerically in [11] and explicitly in