International Journal of Mechanical Sciences 65 (2012) 1–11

Contents lists available at SciVerse ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

A simplified analytical model for post-processing experimental results from tube bulging test: Theory, experimentations, simulations

Nathalie Boudeau [a,b,*], Pierrick Malécot [a,b]

[a] ENSMM, 26 rue Epitaphe, 25030 Besançon Cedex, France
[b] FEMTO-ST, Department of Applied Mechanics, 24 rue Epitaphe, 25000 Besançon, France

A R T I C L E I N F O

Article history:
Received 24 June 2011
Received in revised form
27 July 2012
Accepted 1 August 2012
Available online 10 September 2012

Keywords:
Tube bulging test
Analytical model
FE simulations
Hill 1948
Hill 1993
316L stainless steel

A B S T R A C T

A complete analytical model combined with a very simple experimental procedure is proposed. It permits the post-processing of experimental measures to obtain the stress–strain curve for tubes very quickly and well adapted for industrial use. The quality of the results is proved by comparison with experimental measures and finite element results. Anisotropy in tube is revealed by plotting the (ρ,α) curve where ρ and α stand for strain and stress path respectively. Two quadratic criteria (Hill 1948 and Hill 1993) are studied and it is found that the Hill 1993 criterion seems the best to represent tube anisotropy for 316L stainless steel tube studied in the present paper.

1. Introduction

Tube hydroforming process consists in forming a tube inside a closed and shaped cavity by an internal pressure. For complex shapes and thinning limitation, a combination of an internal pressure and a compression axial force is needed. This technology presents a great industrial interest because it permits to obtain complex hollow shaped parts with a reduced number of welding spots and higher structural quality [1–4]. This process is particularly developed in competitive industries where finite element simulations are intensively used to decrease lead time for design. For that, efficient FE models are needed and it is well known that material data still represents a critical point. Too often, simulations are based on material characteristics obtained from tensile test done on flat sheet specimen. These material data present several limitations: (1) for a same material grade, one cannot compare a flat sheet with a tube; (2) engineer strains are limited to around 20% due to necking that is very low compared to the deformation possibilities for the loading conditions in the hydroforming process; (3) for advanced steels (like TRiP steels for example), plastic behaviour strongly depends on strain path.

By analogy with bulge test for sheets, the tube bulging test is recommended for material characterisation dedicated to tube hydroforming. A tube clamped at its two extremities is put under an internal pressure and freely expands along a called "free zone" (Fig. 1).

To get material data from these tests, it is necessary to develop specific model and no standard is defined at the present time [5]. So several authors have proposed different approaches for the experimental data post-processing that can be classified into three families: (1) approaches based on 'off-line' measurements [6,7], (2) approaches based on 'on-line' measurements [8–11] and (3) approaches based on a mix of 'on-line' and 'off-line' measurements [12,13]. The first and third families are not satisfying because the approaches are very time and material consuming. Some models are based on strong assumptions such as hardening law [6,12], thickness evolution [9,10]. Other approaches need FE simulations and iterative methods [6,9].

Moreover, the tube bulging test is quite complex and several sources of uncertainty exist. It is then important to be able to quantify the uncertainty on the resulting material data. For that rapid procedure is needed if global sensitivity method is planned to be conducted.

In the present paper, an evolution of the Velasco and Boudeau model [11] is proposed. In [11] a semi-analytical model was suggested. The resolution method was based on a Newton–Raphson algorithm. Therefore, it is not well adapted for the evaluation of errors on the resulting hardening curve. So a

* Corresponding author at: ENSMM, 26 rue Epitaphe, 25030 Besançon Cedex, France. Tel.: +33 381 666 034; fax: +33 381 666 700.
E-mail addresses: Nathalie.Boudeau@ens2m.fr (N. Boudeau), Pierrick.Malecot@ens2m.fr (P. Malécot).

0020-7403/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijmecsci.2012.08.002

complete analytical modelling combined with a very simple experimental procedure is proposed in Section 2 and provide an experimental method adapted to industrial context to get the strain–stress curve. In Section 3 experimental and numerical works are presented. Results and discussion can be found in Section 4.

2. Theory

2.1. Geometrical representation

The first step for the establishment of the analytical model is a geometrical representation of a bulged tube. From the observation done on FE simulations of the tube bulging test, it permits to postulate that its longitudinal profile can be approached by an arc of circumference. So the parameterisation described in Fig. 2 can be proposed. The definition of the different parameters is given in Table 1.

From Fig. 2, the following geometrical parameters can be evaluated:

$$R = \frac{h^2 + d^2}{2h} \tag{1}$$

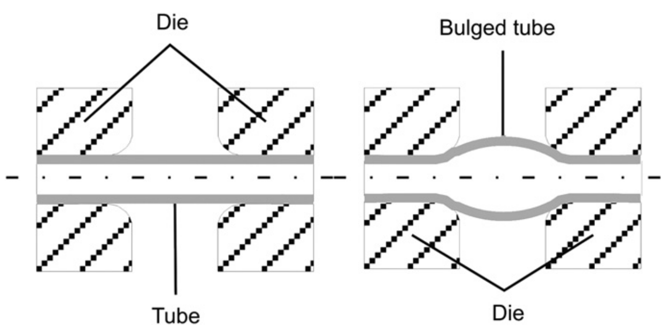

Fig. 1. The tube bulging test: a schematic description.

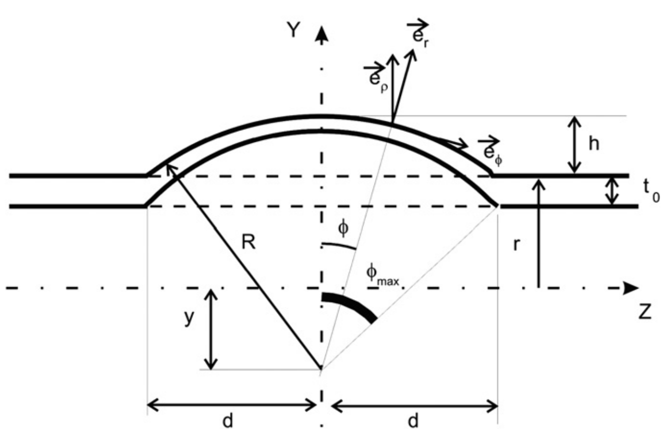

Fig. 2. Parameterisation of the tube bulging test.

Table 1
List of the parameters used for the tube bulging test representation.

r	Initial tube radius (given)
d	Half length of free bulging (given)
t_0	Initial tube thickness (given)
h	Bulging height (measured)
R	Radius of the arc of circumference (calculated)
y	Position/coordinate of the centre of the arc of circumference (calculated)
ϕ_{max}	Angular sector of the arc of circumference (calculated)

$$y = r + h - R < 0 \tag{2}$$

$$\sin(\phi_{\max}) = \frac{d}{R} \tag{3}$$

It is also possible to calculate the coordinates for each point of the tube profile

$$Z(\phi) = R\sin\phi \tag{4}$$

$$Y(\phi) = y + \sqrt{R^2 - Z(\phi)^2} \tag{5}$$

2.2. Strains calculation

For strains evaluation a local frame $(\overrightarrow{e_\phi}, \overrightarrow{e_\theta}, \overrightarrow{e_r})$ is defined as illustrated in Fig. 3. In this set of axis the strain tensor takes the following form:

$$\underline{\varepsilon}(M) = \begin{pmatrix} \varepsilon_{\phi\phi}(\phi) & 0 & 0 \\ 0 & \varepsilon_{\theta\theta}(\phi) & 0 \\ 0 & 0 & \varepsilon_{rr}(\phi) \end{pmatrix}_{(\overrightarrow{e_\phi}, \overrightarrow{e_\theta}, \overrightarrow{e_r})} \tag{6}$$

where $\varepsilon_{\phi\phi}$ is the true longitudinal strain in the tube, $\varepsilon_{\theta\theta}$ the true circumferential strain and ε_{rr} the true radial strain.

They can be calculated as following:

$$\varepsilon_{\phi\phi}(\phi) = \ln\left(\frac{R\phi_{\max}}{d}\right) \tag{7}$$

$$\varepsilon_{\theta\theta}(\phi) = \ln\left(\frac{Y(\phi)}{r}\right) \tag{8}$$

then $\varepsilon_{\phi\phi}$ does not depend on ϕ and is constant along the tube length.

The last strain tensor component can be evaluated by using the incompressibility condition

$$\varepsilon_{rr}(\phi) = -\varepsilon_{\phi\phi}(\phi) - \varepsilon_{\theta\theta}(\phi) \tag{9}$$

Finally, from Eq. (9) the current thickness can be calculated

$$t(\phi) = t_0 \exp[\varepsilon_{rr}(\phi)] \tag{10}$$

2.3. Stress calculation

In the same set of axis, by considering thin tube, the stress tensor can be expressed as follows:

$$\underline{\sigma}(M) = \begin{pmatrix} \sigma_{\phi\phi}(\phi) & 0 & 0 \\ 0 & \sigma_{\theta\theta}(\phi) & 0 \\ 0 & 0 & 0 \end{pmatrix}_{(\overrightarrow{e_\phi}, \overrightarrow{e_\theta}, \overrightarrow{e_r})} \tag{11}$$

For the evaluation of stress components, equilibriums of elementary volumes of tube under pressure are studied. The two infinitesimal parts of tube considered for these evaluations are given in Fig. 4. From these mechanical equilibriums the two following equations are obtained:

$$t(\phi)Y(\phi)\cos(\phi)\sigma_{\phi\phi}(\phi) - \tfrac{1}{2}p[Y(\phi)]^2 = C \tag{12}$$

$$\frac{\sigma_{\theta\theta}(\phi)}{Y(\phi)} + \cos(\phi)\frac{\sigma_{\phi\phi}(\phi)}{R} = \frac{p}{t(\phi)} \tag{13}$$

where p is the internal pressure inside the tube and C a constant.

To obtain the stress tensor components, it is essential to get a value for the constant C. For that, the mechanical equilibrium of a half longitudinal slice of tube with no thickness is considered (Fig. 5). It gives the following equations:

$$\begin{cases} \sigma_{\phi\phi}(\phi_{\max})t(\phi_{\max})\sin(\phi_{\max}) = pd \\ \sigma_{\phi\phi}(0)t(0) = ph + \sigma_{\phi\phi}(\phi_{\max})t(\phi_{\max})\cos(\phi_{\max}) \end{cases} \tag{14}$$