International Journal of Non-Linear Mechanics 45 (2010) 382–394

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Parametric vibrations and stability of an axially accelerating string guided by a non-linear elastic foundation

Mergen H. Ghayesh *

Department of Mechanical Engineering, McGill University, 817 Sherbrooke St. West, Montreal, Quebec, Canada H3A 2K6

ARTICLE INFO

Article history:
Received 19 March 2009
Received in revised form
3 December 2009
Accepted 28 December 2009

Keywords:
Non-linear vibration
Method of multiple scales
Bifurcation

ABSTRACT

Parametric vibrations and stability of an axially accelerating string guided by a non-linear elastic foundation are studied analytically. The axial speed, as the source of parametric vibrations, is assumed to involve a mean speed, along with small harmonic variations. The method of multiple scales is applied to the governing non-linear equation of motion and then the natural frequencies and mode shape equations of the system are derived using the equation of order one, and satisfying the compatibility conditions. Using the equation of order epsilon, the *solvability conditions* are obtained for three distinct cases of axial acceleration frequency. For all cases, the stability areas of system are constructed analytically. Finally, some numerical simulations are presented to highlight the effects of system parameters on vibration, natural frequencies, frequency–response curves, stability, and bifurcation points of the system.

1. Introduction

Problems classified as axially moving systems can be simplified models of many technological devices such as band saw blades, aerial tramways, textile fibers, magnetic tapes, and conveyor belts. In spite of many advantages of these kinds of engineering devices, vibration and noise that arise in these systems have limited their mechanical applications. For example, in a conveyor belt system, vibration of the belt causes noise and wear in the system. Therefore, understanding vibrations and stability characteristics of axially moving systems is of great importance for the optimal design of such engineering devices.

In axially moving systems, vibrational behavior is often the main factor that affects the mechanical design of the system. On increasing the axial speed, the natural frequency decreases, and the system reaches a certain speed, named as the *critical speed*, at which one of the natural frequencies of the system vanishes and afterwards, the system may experience instability and severe vibrations. Instability can also occur at sub-critical traveling speed for systems subjected to a time-varying speed (as the main source of parametric vibrations), even though the mean speed is in the low scales. Therefore, prediction of vibrational behavior of the system in both sub- and super-critical ranges of axial speed is of great concern for optimal design of these types of systems.

Either a beam model [1–6] or a string model [7–9] of axially moving materials can be used to analyze the vibrational behavior of such systems. For example, the non-linear vibration of a traveling tensioned beam in sub- and super-critical speed ranges was studied by Wickert [1]. Finite element method was used by Stylianou and Tabarrok [2] to obtain a numerical solution of the equation governing the motion of an axially moving beam. The stability analysis of an axially moving beam was studied by Stylianou and Tabarrok [3] using the finite element method. The vibration of an axially moving beam with the assumption of weak non-linearities was investigated by Pellicano and Zirilli [4]. Chakraborty et al. [5] investigated free and forced vibration of a non-linear traveling slender beam. Marynowski and Kapitaniak [6] studied the vibration of a Zener damping model of an axially moving visco-elastic beam. Parker [7] studied the stability of an axially moving string supported by a discrete elastic foundation. The vibration of an axially moving strip with edge imperfection and guided by a partial elastic foundation was investigated by Kartik and Wickert [8].

Either a linear model [7,8] or a non-linear one [1,4,5] can be considered for both string and beam models of axially moving systems. When the amplitude is large, predictions from the linear theory are unreliable, and subsequently the use of a non-linear model is essential.

* Tel.: +1 514 398 6290.
E-mail address: mergen.hajghayesh@mail.mcgill.ca

0020-7462/$ - see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijnonlinmec.2009.12.011

The axial speed can also be considered as a constant value [1,4,7], with the aim of investigating the super-critical dynamics, or a time-variant value [10–12] to investigate the parametric resonances. Pakdemirli and Oz [10] studied the transverse vibrations of a simply supported axially moving beam. In their work, traveling beam eigenfunctions with infinite number of modes were considered. Non-linear transversal vibrations and the stability of an axially moving visco-elastic string supported by a linear visco-elastic guide were studied by Ghayesh [11]. The parametric vibration of an axially moving visco-elastic Rayleigh beam was investigated by Ghayesh and Balar [12]. For further review of the vast literature, one can see Ref. [13].

In this study, parametric vibrations and stability of an axially accelerating string guided by a partial non-linear elastic foundation are investigated analytically. The method of multiple scales is applied to the non-linear equation of motion and for three distinct cases of axial acceleration frequency, the solvability conditions are obtained and the stability boundaries are developed analytically. Eventually, the effects of system parameters such as foundation length and location, mean velocity, and linear and non-linear stiffness coefficients of the foundation on natural frequencies, frequency–response curves, and bifurcation points of the system are investigated through parametric studies.

2. Model development

Consider a simply supported string guided by a non-linear elastic foundation, which travels at the speed of $v(t)$ (Fig. 1). The string is considered as a three-part system, i.e. $0<x<a$, $a<x<(a+b)$, which is subjected to a non-linear elastic foundation, and $(a+b)<x<(a+b+c)$.

The foundation reaction force, F, and boundary conditions for a simply supported string are in the form

$$F=k_1u+k_3u^3, \tag{1a}$$

$$\zeta(0)=0, \quad \zeta(1)=0, \tag{1b}$$

where u, k_1, and k_3 are, respectively, the transverse displacement, the linear and non-linear stiffness coefficients of the foundation per unit length, and ζ is the dimensionless transverse displacement.

The equation of motion for spans not subjected to the foundation is [11]

$$\frac{\partial^2\zeta}{\partial T^2}+\frac{dc_v}{dT}\frac{\partial\zeta}{\partial\eta}+2c_v\frac{\partial^2\zeta}{\partial\eta\,\partial T}+(c_v^2-1)\frac{\partial^2\zeta}{\partial\eta^2}=\frac{3}{2}\mu^2\frac{\partial^2\zeta}{\partial\eta^2}\left(\frac{\partial\zeta}{\partial\eta}\right)^2, \tag{2}$$

in which

$$\zeta=u/(a+b+c),\eta=x/(a+b+c),T=t\sqrt{p/\rho A(a+b+c)^2},c_v(T)=v\sqrt{\rho A/p},\mu=\sqrt{EA/p}, \tag{3}$$

where A is the cross-sectional area of the string, E the Young's modulus, p the pretension in the string, ρ the density of the string, and u the transverse displacement.

The equation of motion for the span subjected to the non-linear elastic foundation is

$$\frac{\partial^2\zeta}{\partial T^2}+\frac{dc_v}{dT}\frac{\partial\zeta}{\partial\eta}+2c_v\frac{\partial^2\zeta}{\partial\eta\,\partial T}+(c_v^2-1)\frac{\partial^2\zeta}{\partial\eta^2}+\alpha_1\zeta+\alpha_3\zeta^3=\frac{3}{2}\mu^2\frac{\partial^2\zeta}{\partial\eta^2}\left(\frac{\partial\zeta}{\partial\eta}\right)^2, \tag{4}$$

where

$$\alpha_1=k_1(a+b+c)^2/p,\alpha_3=k_3(a+b+c)^4/p. \tag{5}$$

Considering Heaviside function, and using Eqs. (2) and (4), the equation of motion of the entire string takes the form

$$\frac{\partial^2\zeta}{\partial T^2}+\frac{dc_v}{dT}\frac{\partial\zeta}{\partial\eta}+2c_v\frac{\partial^2\zeta}{\partial\eta\,\partial T}+(c_v^2-1)\frac{\partial^2\zeta}{\partial\eta^2}+[H(\eta-\overline{a})-H(\eta-\overline{a}-\overline{b})](\alpha_1\zeta+\alpha_3\zeta^3)=\frac{3}{2}\mu^2\frac{\partial^2\zeta}{\partial\eta^2}\left(\frac{\partial\zeta}{\partial\eta}\right)^2, \tag{6}$$

where $\overline{a}=a/(a+b+c)$ and $\overline{b}=b/(a+b+c)$.

Assuming that the axial speed involves a mean speed along with small harmonic variations, one has

$$c_v(T)=\overline{c}+\varepsilon c_0\sin\alpha T, \tag{7}$$

in which the $\overline{c}$ is the mean speed, α the frequency of the axial speed, and εc_0 the amplitude of small variations.

3. Solution using a perturbation technique

In this section, an approximate analytical solution will be obtained using the method of multiple scales; which is an important perturbation technique.

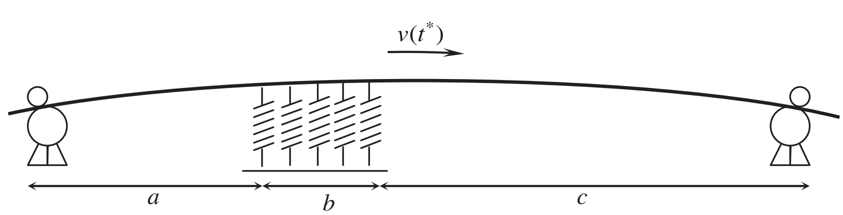

Fig. 1. An axially accelerating string on a non-linear elastic foundation.