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Dynamic stability and instability of a double-beam system subjected to random forces

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ABSTRACT

On the basis of the Bernoulli–Euler beam theory, the stability and instability of a double-beam system subjected to compressive axial loading is investigated. It is assumed that the two beams of the system are simply supported and continuously joined by a Winkler elastic layer. Each pair of axial forces consists of a constant part and a time-dependent stochastic function. By using the direct Lyapunov method, bounds of the almost sure stability and instability and uniform stochastic stability of a double-beam system as a function of viscous damping coefficient, bending stiffness, stiffness modulus of the Winkler layer, variances of the stochastic forces, and intensity of the deterministic components of axial loading are obtained. When the almost sure stability and instability are investigated, numerical calculations are performed for the Gaussian process with a zero mean as well as a harmonic process with random phase. When axial forces are white noise processes, conditions for uniform stochastic stability are determined.

1. Introduction

Beam-type structures are widely used in many branches of civil, mechanical and aerospace engineering. The dynamic problems of single beams based on Bernoulli–Euler theory have been studied by many researchers.

An important technological extension of the concept of the single beam is that of the elastically connected double-beam system. Such a system is another model of a complex continuous system consisting of two one-dimensional solids joined by a linear, elastic layer of Winkler type. Various problems of double-beam systems occupy an important place in many fields of structural and foundation engineering. In many of the soil–structure interaction problems, the elastic foundation has been usually modelled by a Winkler elastic layer. It is also known that one beam and elastic layer of a double-beam system can be considered as a continuous dynamic absorber to suppress the vibration of another beam subjected to a dynamic force. Elastically connected beams are used by Chen and Sheu [1] as an approximate model for vibration analyses of composite materials or by Ru [2] as continuous system models for carbon nanotubes. The elastic layers provide a linear model for inter-atomic Van der Waals forces.

Having in mind the possibly wide applications in various areas of technology, the dynamics of the double-beam system is a subject of great interest. With arbitrary boundary conditions and forcing functions, the problem is difficult to solve because the governing partial differential equations are coupled. However, under certain conditions, the problem becomes trackable.

Natural frequencies and buckling stresses of a deep beam–column on two-parameter elastic foundations by taking the effect of shear deformations, depth change and rotatory inertia are analysed by Matsunga [3]. The vibration of a double-beam system consisting of a main beam with an applied force, and an auxiliary beam with a distributed spring and dashpot in parallel between the two beams, is studied by Vu et al. [4]. Seelig and Hoppman II [5] present the development and solution of the differential equations of motion of an elastically connected double-beam system subjected to an impulsive excitation. The main beam is subjected to a completely arbitrary forcing function which can be either concentrated at any point or distributed. The complete exact theoretical solutions of the free vibrations of simply supported Bernoulli–Euler double-beam system are treated by Oniszczuk [6]. The eigenfrequencies and mode shapes of vibration of the considered double-beam system are found using the classical assumed mode summation. Also, the presented theoretical analysis is illustrated by a numerical example, in which the effect of physical parameters characterizing the vibrating system on the natural frequencies is investigated. The dynamic response of a double-beam system traversed by a constant moving load is studied by Abu-Hilal [7]. The effect of the moving speed of the

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load and the damping and the elasticity of the viscoelastic layer on the dynamic responses of the beams are investigated in detail. The free vibration frequencies of beams system having to the ends translation and rotation elastic constraints are determined by De Rosa and Lippiello [8]. However, these studies are limited to the cases with negligible axial loads.

The free vibration and buckling of a double-beam system under deterministic axial loading are studied by Zhang et al. [9]. It is supposed that the buckling can only occur in the plane where the double-beam system lies. The effects of compressive axial load on the properties of forced transverse vibration of a double-beam system are investigated by Zhang et al. [10]. A general theory for the determination of natural frequencies and mode shapes for a set of elastically connected axially loaded Bernoulli–Euler beams is developed by Kelly and Srinivas [11]. In the special case of identical beams, it is shown that the natural frequencies are organized into sets of intramodal frequencies in which each mode shape is a product of a spatial mode and a discrete mode. When studying the equation for a single axially loaded beam, numerical difficulties arise in the determination of natural frequencies due to the presence of exponentially large terms, as is noted by Williams [12]. Stochastic stability of a double-beam system subjected to small intensity white noise excitation is investigated by Kozić et al. [13].

The purpose of the present paper is the investigation of the almost sure stability and instability as well as uniform stochastic stability of a double-beam system as a function of damping coefficients, variances of the stochastic forces, bending stiffnesses, stiffness modulus of the Winkler layer and intensity of the deterministic components of axial loading. The principal contribution of this paper is to clearly fix the boundaries of stability, uncertainty and instability regions when the system is loaded at one or both beams.

The present paper is organized as follows. For the governing differential equations, the definition of almost-sure and uniform stochastic stability is given in Section 2, the Lyapunov functional is constructed in Section 3 as a measure of distance between the solution and the trivial one. The conditions of almost-sure stability and instability are obtained in Sections 4 and 5, while the condition for uniform stability is obtained in Section 6. The numerical procedure of determining the boundaries of stability and instability, as well as the analysis of obtained results, is given in Section 7. Section 8 ends the paper with concluding remarks.

2. Problem formulation

Let us consider the system which is composed of two parallel, slender, prismatic, and homogeneous beams continuously joined by a Winkler elastic layer. Both beams have the same length and are simply supported at their ends. The beams are subjected to axial compressions F_1 and F_2 as is shown in Fig. 1.

On the basis of the Bernoulli–Euler theory, and assuming that both the rotary inertia and shear deformation are negligible, the coupled governing differential equations for transverse vibrations

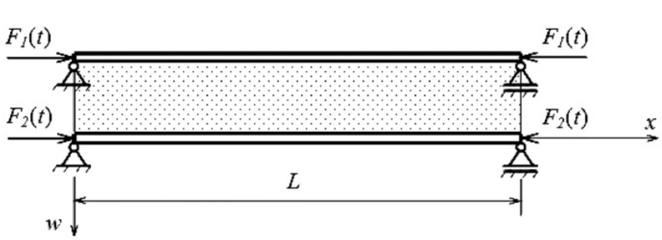

Fig. 1. The physical model of an elastically connected double-beam system.

of the system can be expressed by

$$\rho_1 A_1 \frac{\partial^2 w_1}{\partial t^2} + c_1 \frac{\partial w_1}{\partial t} + E_1 I_1 \frac{\partial^4 w_1}{\partial X^4} + F_1(t) \frac{\partial^2 w_1}{\partial X^2} + \overline{K}(w_1 - w_2) = 0, \quad (1)$$

$$\rho_2 A_2 \frac{\partial^2 w_2}{\partial t^2} + c_2 \frac{\partial w_2}{\partial t} + E_2 I_2 \frac{\partial^4 w_2}{\partial X^4} + F_2(t) \frac{\partial^2 w_2}{\partial X^2} + \overline{K}(w_2 - w_1) = 0, \quad (2)$$

where w_i $(i=1,2)$ denotes the transverse beam displacement, ρ_i is the mass density, c_i is the viscous damping coefficient, X is the axial coordinate, t is the time, $E_i I_i$ is the bending stiffness of the beam, $\overline{K}$ is the stiffness modulus of the Winkler elastic layer, and $F_1(t)$,$F_2(t)$ are time-dependent stationary stochastic processes.

Boundary conditions corresponding to simply supported edges have the form:

$$\left.\begin{matrix} X=0 \\ X=L \end{matrix}\right\}, \quad w_1 = 0, \quad \frac{\partial^2 w_1}{\partial X^2} = 0, \quad w_2 = 0, \quad \frac{\partial^2 w_2}{\partial X^2} = 0, \quad (3)$$

where L is the length of the beams.

Let us assume that products $\rho_1 A_1$ and $\rho_2 A_2$ are equal ($\rho_1 A_1 = \rho_2 A_2 = \rho A$) but the individual parameters can be arbitrary. The following parameters can be used to non-dimensionalize equations (1):

$$X = Lx, \quad 2\beta_i = \frac{c_i}{\rho A}, \quad e_i = \frac{E_i I_i}{\rho A L^4}, \quad f_{oi} + f_i(t) = \frac{F_i(t)}{\rho A L^2}, \quad K = \frac{\overline{K}}{\rho A} \quad (i=1,2) \quad (4)$$

where β_i, e_i and K are damping coefficient, reduced stiffness and reduced stiffness of the Winkler layer, respectively, f_{oi} and $f_i(t)$ are reduced constant and stochastic component of axial forces.

Now, they have the form

$$\frac{\partial^2 w_1}{\partial t^2} + 2\beta_1 \frac{\partial w_1}{\partial t} + e_1 \frac{\partial^4 w_1}{\partial x^4} + (f_{o1} + f_1(t)) \frac{\partial^2 w_1}{\partial x^2} + K(w_1 - w_2) = 0, \quad (5)$$

$$\frac{\partial^2 w_2}{\partial t^2} + 2\beta_2 \frac{\partial w_2}{\partial t} + e_2 \frac{\partial^4 w_2}{\partial x^4} + (f_{o2} + f_2(t)) \frac{\partial^2 w_2}{\partial x^2} + K(w_2 - w_1) = 0. \quad (6)$$

The purpose of the present paper is the investigation of almost sure asymptotic and uniform stochastic stability of the double-beam system subjected to stochastic time-dependent axial loads. To estimate perturbated solutions, it is necessary to introduce a measure of distance $||\cdot||$ of solutions of Eqs. (5) and (6) with non-trivial initial conditions and the trivial one. Following Kozin [14], the equilibrium state of Eqs. (5) and (6) is said to be almost surely stochastically stable, if:

$$P\left\{\lim_{t\to\infty} \|\mathbf{w}(.,t)\| = 0\right\} = 1, \quad (7)$$

where $\mathbf{w} = col(w_1, w_2)$ is the matrix column.

In the case when the loadings are broad-band Gaussian processes which can be treated as white noises, we investigate the uniform stochastic stability of the trivial solution, i.e. we formulate conditions implying the following logical sentence:

$$\bigwedge_{\varepsilon>0} \bigwedge_{\delta>0} \bigvee_{t>0} ||\mathbf{w}(.,t)|| < r \Rightarrow P\left\{\sup_{t>0} ||\mathbf{w}(.,t)|| > \delta\right\} < \varepsilon. \quad (8)$$

3. Lyapunov functional construction

The problem formulated in the previous section we will solve by using the Lyapunov functional method. One of the first general methods of constructing Lyapunov functional for deterministic systems was given by Parks and Pritchard [15]. Plaut and Infante [16] provided the construction of Lyapunov functional for continuous systems subjected to random excitation, while Kozin [14] introduced the best functional. For the study of stability of