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Complete equilibrium paths for Mises trusses

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ABSTRACT

This paper presents determination of equilibrium paths for Mises trusses with different ratio of height to span. Unsymmetrical deformation modes are considered and the structure is treated as a two DOF system. First, a few special equilibrium configurations are resolved from considerations of free body diagrams. Complete equilibrium paths are determined by solving numerically the governing non-linear equilibrium equations. The stability of possible equilibrium configurations is checked using the second partial derivative test for the total potential energy. The positive definiteness of the appropriate Hessian matrices is checked numerically using the Sylvester criterion.

1. Introduction

It is a typical feature of instability theory that its fundamental characteristics can be analyzed using very simple models, with one or two degrees of freedom (DOFs) [1,2]. Moreover, even a very simple model based on unrealistic assumptions such as perfect unlimited material elasticity can give us surprisingly complex results due to large displacements and deformations. Many often such simple example problems provide good insight into the stability of large structural systems [3].

Considered here Mises truss [4,5] shown in Fig. 1 is an example of a classical elastic system having numerous references in the literature. Under assumption that the truss is sufficiently shallow and deflects only symmetrically, it is usually treated as a single DOF system to analyze the problem of "snap-buckling". Although the system is simple it features the buckling phenomenon of more complex structures such as slightly curved membranes and thin spherical shells [6]. As the one DOF, symmetrical system it has been analyzed statically and dynamically in numerous variations, with added rotational and longitudinal springs [6], dampers or main members subject to bending [7]. However, there is little material available considering steep trusses which are allowed to deform asymmetrically, and some of such solutions [8] obtained with aid of finite element (FE) method are qualitatively incorrect to some extent. The aim of this paper is to determine all possible equilibrium paths of a Mises truss as a two DOF system. Location of the limit and bifurcation points as well as determination of the stable and unstable equilibrium states is also under the consideration.

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2. Description and notation

The considered system, shown in Fig. 1, consists of two equal pin-jointed straight bars (or springs) sustaining only axial loading (infinite flexural stiffness). The system is loaded with vertical, conservative force $P=\lambda EA_0$, where λ is the loading parameter and EA_0 is the member's axial stiffness. Fig. 1 provides the notation of geometrical variables applied in the following text and describing the initial and deflected configurations. It is assumed that the members are elastic and deform uniformly, and consequently the axial forces are proportional to the axial strains. Only displacements within the xy plane are allowed. Here, all possible equilibrium states, also unsymmetrical, are sought without any restriction upon initial configuration.

For large deformations a convenient finite strain measure is the Green–Lagrange [9] which, for axially deformed member, can be expressed in local coordinate system as

$$\varepsilon_{\xi\xi}=\frac{\partial u_\xi}{\partial \xi}+\frac{1}{2}\left(\frac{\partial u_\xi}{\partial \xi}\right)^2, \tag{1}$$

where ξ is the local coordinate along the member's axis and $u_\xi(\xi)$ *is* the axial displacement field. Assuming uniform deformation along the member we get

$$u_\xi(\xi)=\frac{l-l_0}{l_0}\xi \rightarrow \varepsilon_{\xi\xi}=\frac{l^2-l_0^2}{2l_0^2}, \tag{2}$$

where l denotes current length. Finally the internal force induced in an i-member with stiffness EA_0 is given by

$$N_i=EA_0\frac{l_i^2-l_0^2}{2l_0^2}. \tag{3}$$

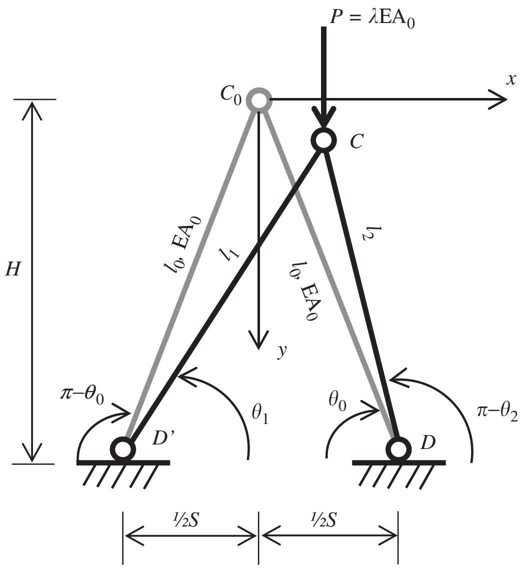

Fig. 1. Mises truss with two DOF.

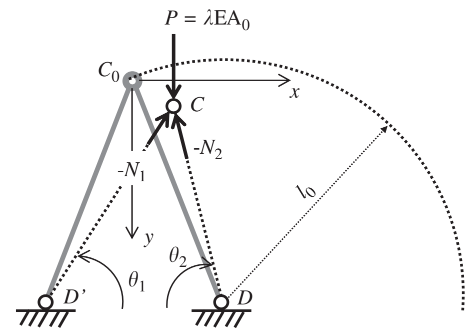

Fig. 2. Free-body diagram showing equilibrium for both bars contracted.

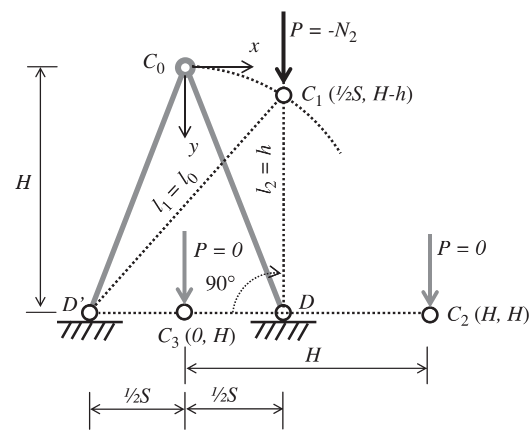

Fig. 3. Location of two characteristic equilibrium points C_1 and C_2.

The following geometrical relations with notation presented in Fig. 1 are used in the proceeding text:

$$s_0 = \sin\,\theta_0 = \frac{H}{l_0},\quad c_0 = \cos\,\theta_0 = \frac{0.5S}{l_0},\quad l_0 = \sqrt{(0.5S)^2 + H^2},$$
$$s_1 = \sin\,\theta_1 = \frac{H-y}{l_1},\quad c_1 = \cos\,\theta_1 = \frac{0.5S+x}{l_1},$$
$$l_1 = \sqrt{(0.5S+x)^2 + (H-y)^2},$$
$$s_2 = \sin\,\theta_2 = \frac{H-y}{l_2},\quad c_2 = \cos\,\theta_2 = \frac{0.5S-x}{l_2},$$
$$l_2 = \sqrt{(0.5S-x)^2 + (H-y)^2}. \tag{4}$$

3. Selected equilibrium points

Fig. 2 shows the free-body diagram for the hinge C_0, displaced to an arbitrary position C. It illustrates equilibrium for two contracted bars (with negative internal forces)

$$\sum X:\ p_x = -N_1c_1 + N_2c_2 = 0,$$
$$\sum Y:\ p_y = P + N_1s_1 + N_2s_2 = 0. \tag{5}$$

Using (3) and (4) the equilibrium equations (5) can be written in a different form as

$$-(l_1^2 - l_0^2)c_1 + (l_2^2 - l_0^2)c_2 = 0,$$
$$2\lambda l_0^2 + (l_1^2 - l_0^2)s_1 + (l_2^2 - l_0^2)s_2 = 0. \tag{6}$$

The objective here is to find real roots of Eqs. (6) indicating locations of possible equilibrium and to check if they are stable or not. Regardless of the selection of generalized coordinates as (θ_1, θ_2), (l_1, l_2) or (x, y), Eqs. (6) are highly non-linear and it is difficult to find their roots in a closed form.

First we try to determine a few locations satisfying (6). Fig. 3 shows three characteristic equilibrium locations C_1, C_2 and C_3, in the first quadrant and Eqs. (7) give the corresponding solution parameters

$$\text{for } C_1:\ l_1 = l_0,\quad l_2 = h,\quad h = \sqrt{H^2 - \tfrac{3}{4}S^2},\quad N_1^{C_1} = 0,$$
$$N_2^{C_1} = \frac{-2S^2}{4H^2 + S^2}EA_0,$$
$$\lambda^{C_1} = \frac{2S^2}{4H^2 + S^2} = \frac{2}{\tan^2\theta_0 + 1},$$
$$\text{for } C_2:\ l_1 = \tfrac{1}{2}S + H,\quad l_2 = -\tfrac{1}{2}S + H,$$
$$N_1^{C_2} = -N_2^{C_2} = \frac{2HS}{4H^2 + S^2}EA_0,\quad \lambda^{C_2} = 0,$$
$$\text{for } C_3:\ N_1^{C_3} = N_2^{C_3} = \frac{-2H^2}{4H^2 + S^2}EA_0,\quad \lambda^{C_3} = 0. \tag{7}$$

For the point C_1 we have vertical equilibrium between the load and the internal force in the contracted member l_2. For the point C_2 there is a horizontal equilibrium between internal forces in both members (tension and compression) with zero loading. Similarly for the point C_3 the loading is equal to zero but now both members are sustaining the same contraction. The presence of the points C_1 and C_2 is limited to the cases where the second member's length is non-negative. The limit deflected positions are those where (theoretically) l_2 is reduced to zero

$$\text{for } C_1\ :\ l_2 = h \geqslant 0 \Rightarrow H \geqslant \tfrac{\sqrt{3}}{2}S \quad\text{and}\quad \theta_0 \geqslant 60^\circ.$$
$$\text{for } C_2\ :\ l_2 = H - \tfrac{1}{2}S \geqslant 0 \Rightarrow H \geqslant \tfrac{1}{2}S \quad\text{and}\quad \theta_0 \geqslant 45^\circ. \tag{8}$$

Inequalities (8) lead us to an interesting result for the truss with $\theta_0 = 60^\circ$, with shown in Fig. 4 where both points C_1 and C_2 lay on