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The effects of the side walls on the flow of a second grade fluid in ducts with suction and injection

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Abstract

The effects of the side walls on the flow in ducts with suction and injection are examined. Three illustrative examples are given. The first example considers the effect of the side walls on the flow over a porous plate. The second example considers the flow between two parallel porous plates and the third example is devoted to the investigation of the flow in a rectangular duct with two porous walls. Exact solution of the governing equation using the no-slip boundary condition and an additional condition are obtained. The expression of the velocity, the volume flux and the vorticity are given. It is found that for large values of the cross-Reynolds number near the suction region the flow for a Newtonian fluid does not satisfy the boundary condition, but it does not behave in the same way for a second grade fluid. Three examples considered show that there are pronounced effects of the side walls on the flows of a second grade fluid in ducts with suction and injection.

Keywords: Second grade fluid; Non-Newtonian fluid; Exact solution; Effects of the side walls; Flow with suction and injection

1. Introduction

In this paper, some exact solutions of the governing equation of a fluid of second grade are given. Obtaining the exact solutions of the governing equation of a fluid of second grade is very important for many reasons. They provide a standard for checking the accuracies of many approximate methods such as numerical or empirical. Although computer techniques make the complete integration of the governing equation of the fluid of second grade feasible, the accuracy of the results can be established by a comparison with an exact solution. Exact solutions given in this paper are for flows over porous boundaries. The aim of this paper is to investigate the effects of the side walls on the flows over porous plates. In order to show the effects three illustrative examples are given. The first example considers the effects of the side walls on the flow over a porous plate. The second example considers the flow between two parallel walls with uniform injection at one plate and uniform suction at the other. The third example considers the flow in a rectangular duct with impermeable lateral boundaries and the upper and lower boundaries with uniform injection and uniform suction. In order to show the effects of the side walls on the flow of the velocity, the volume flux across a plane normal to the flow and the vorticity are calculated. These three examples show that there are pronounced effects of the side walls on the flow in ducts with suction and injection.

The flow over a porous plane boundary at which there is a uniform suction velocity has been investigated by Griffith and Meredith [1] and they found an exact solution of the Navier–Stokes equations. It can be shown that here is no solution of the Navier–Stokes equation for the flow over a porous plane boundary at which there is a uniform injection velocity. However, if the porous plate is bounded by two side walls, a solution of the Navier–Stokes equations can be obtained for injection case [2]. An extension of the flow for a Newtonian fluid past a porous plate to the flow of a non-Newtonian fluid has been given by many authors [3–6]. In this paper, it is shown that the velocity for a second grade fluid is greater than that of the Newtonian fluid in the absence of the side walls.

It is assumed that a porous plate is bounded by two side walls and the flow over the porous plate is generated by the velocity

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of the porous plate in the absence of the pressure gradient. The solution of the governing equation has three coefficients. The no-slip condition and the condition at infinity determine two of them. In order to determine the third, an additional condition is used. The velocity distribution depends on the ratio of the suction velocity to the plate velocity, the Reynolds number defined by the distance between two side walls and the velocity of the porous plate.

The second illustrative example is the flow between two parallel porous plates with uniform injection at the upper plate and uniform suction at the lower plate. The results for suction at the upper plate and injection at the lower plate can be found without further calculations. When the suction velocity goes to zero, the solution tends to that for the two-dimensional Poiseuille flow. The governing equation is a third order differential equation. The no-slip condition at boundary does not sufficient to determine the solution, therefore, one needs an additional condition. A critical review on the boundary conditions, the existence and uniqueness of the solution has been given by Rajagopal [7]. For large values of the cross-Reynolds number, defined by the suction velocity, the solution for a second grade fluid shows that the velocity vanishes at the upper and lower plates, but the velocity for a Newtonian fluid does not vanish at the suction region near the boundary. The vorticity for a Newtonian fluid is concentrated near the lower plate and it has a constant value across the channel, but it does not behave in the same way for a fluid of second grade.

The third example is the flow in a rectangular duct with uniform suction and injection. This flow for a Newtonian fluid has been examined by Mehta and Jain [8], and Sai and Rao [9] and in [2]. There are some similarities between the flow in a rectangular duct with porous walls and in a circular pipe with porous wall [10]. In this paper, an extension of the flow for a Newtonian fluid in a rectangular duct with porous walls to the flow of a fluid of second grade is considered. Since for a second grade fluid the governing equation in a third order differential equation, to determine the solution one needs an additional condition. It is found that the flow for large values of the cross-Reynolds number, near the suction region, does not satisfy the boundary condition for a Newtonian fluid but it does not behave in the same way for a second grade fluid. The effect of the side walls is greatest for a duct with square cross-section for which the aspect ratio is equal to 1 and when this ratio goes to zero the effect of the side walls disappears. The volume flux across a plane normal to the flow in a rectangular duct with suction and injection for a given value of the cross-Reynolds number decreases approximately linearly with the aspect ratio and for a given value of the aspect ratio decreases with the cross-Reynolds number. For large values of the cross-Reynolds number, the variation of the vorticity for a Newtonian fluid is concentrated near the region of suction and in the other region the vorticity has a constant value, but it does not behave in the same way for a second grade fluid. It is clearly understood from the discussion that the effect of the side walls is very important in practice [11,12]. For practical purposes one wishes to know, for example, how the stress exerted on the bottom wall varies with the distance between the side walls for a given value of the aspect ratio of the channel and if this ratio is large or small, depending on the definition, the stress is nearly uniform and measured value would be unaffected by the presence of the side walls.

2. Basic equations

The equation of motion for a fluid in the absence of body forces is

$$\rho \frac{D\boldsymbol{u}}{Dt} = \nabla \cdot \boldsymbol{\sigma}, \tag{1}$$

where ρ is the density of the fluid, $\boldsymbol{u}$ is the velocity, $\boldsymbol{\sigma}$ is the stress tensor and D/Dt represents the material derivative. The continuity equation is

$$\nabla \cdot \boldsymbol{u} = 0. \tag{2}$$

Eqs. (1) and (2) can be applied to all types of fluids, Newtonian and non-Newtonian. The stress depends on the local properties of the fluid. The relation between the stress and the local properties which is called the constitutive equation is in the following form for a incompressible second grade fluid [13]

$$\boldsymbol{\sigma} = -p\boldsymbol{I} + \mu \boldsymbol{A}_1 + \alpha_1 \boldsymbol{A}_2 + \alpha_2 \boldsymbol{A}_1^2, \tag{3}$$

where μ, α_1 and α_2 are material moduli, $\boldsymbol{A}_n$ represents the Rivlin–Ericksen tensor defined as

$$\boldsymbol{A}_0 = \boldsymbol{I}, \quad \boldsymbol{A}_1 = \nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^{\mathrm{T}},$$

$$A_{n+1} = \left(\frac{\partial}{\partial t} + \boldsymbol{u} \cdot \nabla\right)\boldsymbol{A}_n + (\nabla \boldsymbol{u}) \cdot \boldsymbol{A}_n + [(\nabla \boldsymbol{u}) \cdot \boldsymbol{A}_n]^{\mathrm{T}},$$

where t is time, p is pressure and $\boldsymbol{I}$ is the identity tensor. The Clausius–Duhem inequality and the condition that the Helmholtz free-energy is minimum in equilibrium provide the following restrictions [13–17]

$$\mu \geqslant 0, \quad \alpha_1 + \alpha_2 = 0, \quad \alpha_1 \geqslant 0.$$

The flow of an incompressible fluid of second grade over a porous boundary is considered. The velocity field is assumed to be in the following form:

$$u = u(y, z), \quad v = -V, \quad w = 0, \tag{4}$$

where u, v, w are components of the velocity in rectangular Cartesian coordinates. The x-axis is taken along the plate, the y-axis and the z-axis are perpendicular to the x-axis. Inserting the velocity given by Eq. (4) into the expression of the stress, the components of the stress can be written in the following form:

$$\sigma_{xx} = -p + \alpha_2 \left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right],$$

$$\sigma_{xy} = \mu \frac{\partial u}{\partial y} - \alpha_1 V \frac{\partial^2 u}{\partial y^2},$$

$$\sigma_{yy} = -p + (2\alpha_1 + \alpha_2)\left(\frac{\partial u}{\partial y}\right)^2,$$