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Transport properties in fibrous elastic rhombic composite with imperfect contact condition

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ABSTRACT

In this contribution, effective elastic moduli are obtained by means of the asymptotic homogenization method (AHM), for oblique two-phase fibrous periodic composites with two models (*spring* and *interphase*) of imperfect contact conditions. This work is an extension of previous reported results, where only perfect contact for elastic or piezoelectric composites for square and hexagonal arrays were considered. The constituents of the composites exhibit transversely isotropic properties. A doubly periodic parallelogram array of cylindrical inclusions under longitudinal shear is considered. The behavior of the shear elastic coefficient for different geometry arrays related to the angle of the cell is studied. As validation of the present method, some numerical examples and comparisons with theoretical and experimental results verified that the present model is efficient for the analysis of composites with presence of imperfect interface and parallelogram cell. The effect of the arrangement of the cells on the shear effective property is observed. The present method can provide benchmark results for other numerical and approximate methods.

1. Introduction

The transport properties of circular cylinders packed in regular arrays are of considerable interest in a number of fields. The transport property may be the electrical or thermal conductivities, the dielectric permittivity, or the elastic shear modulus in antiplane elasticity. The result is of interest in the field of materials physics where two phase materials containing rod- or fiber-like inclusions often occur. Knowledge of their electrical or thermal conductivities and their elastic properties is valuable. Calculations on ordered arrays of cylinders are therefore directly relevant to practical situations. The problem of calculating the transport properties will be discussed here in the context of stiffness elastic properties. However, the mathematics and the results obtained are immediately applicable to other associated situations.

In most composites, the fiber–matrix adhesion is imperfect; the continuity conditions for stresses and displacements are not satisfied. Thus various approaches have been used, in which the bond between the reinforcement and the matrix is modeled by an interphase with specified thickness, by Hashin [1], Guinovart-Diaz et al. [2]. Other assumptions suppose that the contrast or jump of the displacements in the interface is proportional to the corresponding component of the tension in the interface in terms of a parameter given by the constant of a spring. This type of imperfect contact (*spring type*) in the interphases of the composites was investigated by Benveniste and Miloh [3] among others and has been used later, for instance, by Achenbach and Zhu [4] and Hashin [5–7].

Molkov and Pobedria [8] reported the elastic effective coefficients for two-phase fibrous composite with rhombic array of periodic cells under perfect contact conditions. Recently, Abolfathi [9] applied a numerical algorithm to determine the homogenized elastic properties of bidirectional fibrous composites and Jiang et al. [10] analyzed different situations of parallelogram composite. In this work, micromechanical analysis method is applied to a periodic composite with unidirectional fibers and parallelogram cells, in particular, rhombic periodic cells. The analytical expressions of the homogenized elastic properties are calculated for two phase composite with imperfect contact conditions. Two approaches (spring and three phase models) are used for the calculation of the shear elastic effective coefficients of angular fibrous composites with anisotropic elastic constituents with no well bonded contact. This contribution is an extension of previous works of Rodriguez-Ramos et al. [11] and Guinovart-Diaz et al. [12] using the asymptotic homogenization method (AHM). The results in this paper are mainly focused on the impact of the fibers cross angles

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and the mechanical imperfection of the interface on the stiffness properties of the chosen composites.

2. General considerations. Contact at the interfaces

Let us consider unidirectional periodic fibers composite with crossing angle θ embedded in a matrix as shown in Fig. 1. The crossing angle of the fibers is assumed to remain constant so that a parallelogram cell with periods w_1, w_2 can be defined. The periodicity of microstructure determines the geometry of the periodic cell S. Thus, a two-phase periodic composite is considered here which consists of a parallelogram array of identical parallel circular cylinders embedded in a homogeneous medium (Fig. 2). As a unidirectional fibrous composite it is assumed that the microstructure of the composite along the third direction (perpendicular to plane of cross-section) remains constant. The fibers are all assumed straight and of circular cross sections with radius R. The material properties of each phase belong to the crystal symmetry class 6 mm, where the axes of material and geometric symmetry are parallel. The composite is not well bonded at the contact between the matrix and the fiber. Therefore, imperfect contact conditions at the interface Γ are considered.

The governing elastic equations for this kind of materials are the Navier equations of linear elasticity. As the body forces are absent, stress ($\boldsymbol{\sigma}$), strain ($\boldsymbol{\varepsilon}$), and displacement ($\boldsymbol{u}=(u_1,u_2,u_3)$) fields satisfy the following three equations, respectively:

$$\text{Stress–strain relations: } \boldsymbol{\sigma}=\mathbf{C}:\boldsymbol{\varepsilon}. \quad (1)$$

$$\text{Displacement-strain relations: } \boldsymbol{\varepsilon}=\frac{1}{2}(\nabla\mathbf{u}+\nabla^{\mathrm{T}}\mathbf{u}). \quad (2)$$

$$\text{Equilibrium equations: } \nabla\cdot\boldsymbol{\sigma}=0, \quad (3)$$

where $\mathbf{C}$ is the elasticity tensor and ∇ is the gradient operator, comma notation is understood to denote differentiation with respect to x_i.

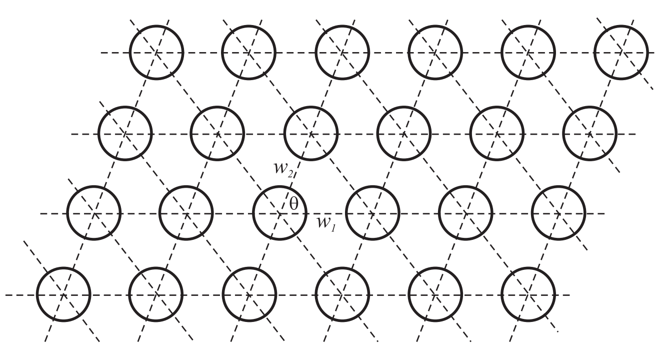

Fig. 1. The cross-section of a rhombic array of angle θ and periods w_1, w_2 of circular fibers.

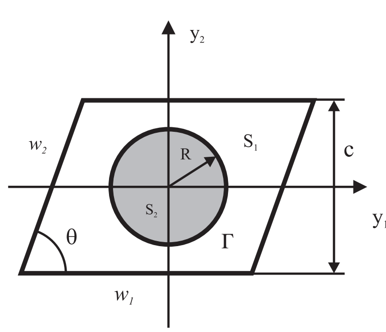

Fig. 2. The unit cell showing the domains S_1 and S_2 occupied by the matrix and fibers materials; Γ is the common interface.

Since most reinforcements may not be perfectly bonded to their surrounding matrix, the perfect bonding condition is often inadequate in describing the physical and mechanical behavior of real composite materials. An imperfect bond may be introduced deliberately by coating the reinforcements to control the properties of the composites and sometimes to improve their fatigue life [1]. Moreover, chemical reactions between reinforcements and the matrix in manufacturing process or the damage caused by cyclic loadings of the composites can develop imperfect bonding interfaces. To model the imperfect bonding at the interfaces, some idealized interfacial conditions have been proposed by various investigators. For example, Gharemani [13], Mura and Furuhashi [14], Mura et al. [15], and Jasiuk et al. [16], among others have used the pure frictionless sliding condition to model grain boundary sliding in polycrystalline materials and particle sliding in soil. Furthermore, the frictional sliding considered by numerous researchers; see, for example, Hashin [1,5], Jasiuk et al. [17], Huang et al. [18], Gao [19], and Zhong and Meguid [20] is a more realistic interfacial condition, in which the frictional resistance of the interface is accommodated by assuming that the discontinuous components of displacement are proportional to the associated tractions

$$\begin{aligned}&\|\sigma_{ij}\|n_j=0,\\&\|u_l\|(\delta_{il}-n_i n_l)=(1/K)T_i, \quad \text{on } \Gamma\\&\|u_l\|n_i n_l=(1/M)N_i\end{aligned} \quad (4)$$

where $T_i=\sigma_{ij}n_j-\sigma_{lm}n_l n_m n_i$ and $N_i=\sigma_{lm}n_l n_m n_i$ are the shear and normal components of the surface traction, respectively. Γ denotes the interface between the fibers and matrix, whereas K and M are values of sliding and debonding parameters. As these parameters become infinite, the perfect bonding condition is recovered, while pure sliding condition occurs when K is zero and M approaches infinity. Since these parameters are related to the macroscopic behavior of composites, the theory suggests that they can be determined by a set of carefully designed experiments.

Since a binary periodic composite is studied, thus, two distinct phases, occupying S_1 and S_2 (Fig. 2) are assumed to be in non-perfect contact along the interface Γ of each cylinder. In order to model various possible damages occurring on the fiber-matrix interface composite two formulations of imperfect bonded are considered as follows:

(i) *Linear spring interface (LSI) model.* A generalized shear lag model [1,5] that can be termed as the mechanically compliant weakly conducting interface is useful for the analysis of the behavior of composites. The imperfect interface proposed is the shear lag model (or the spring layer model): tractions are continuous but displacements are discontinuous across the imperfect interface. The jumps in displacement components are further assumed to be proportional, in terms of the "spring-factor-type" interface parameter, to their respective interface traction components

$$\sigma_t^{(\alpha)}=\widehat{K}||u_t||, \quad \overleftrightarrow{\sigma}^{(1)}\cdot\vec{n}^{(1)}=\overleftrightarrow{\sigma}^{(2)}\cdot\vec{n}^{(2)}, \quad u_n^{(1)}=u_n^{(2)}, \text{ on } \Gamma \quad (5)$$

where the subscripts t and n means tangential and normal directions respectively. $\widehat{K}$ is the proportional interface parameter. The double bar notation is used to denote the jump of the relevant function across the interphase Γ taken from the matrix (1) to the fiber (2) i.e. $\|f\|=f_1-f_2$. Eq. (5) is usually called a weak interface condition. It has been originally proposed by Goland and Reissner [21], later studied by many authors like Benveniste and Miloh [3], Molkov and Pobedria [22], Mahiou and Beakou [23], and Andrianov et al. [24].

(ii) *Interphase contact (IC) model.* The imperfect interface condition is replaced by the explicit three-phase problem of two