Available online at www.sciencedirect.com

ScienceDirect

International Journal of Non-Linear Mechanics 42 (2007) 391–402

INTERNATIONAL JOURNAL OF NON-LINEAR MECHANICS

www.elsevier.com/locate/nlm

A constitutive model for fibrous tissues considering collagen fiber crimp

F. Cacho[a,b], P.J. Elbischger[c], J.F. Rodríguez[b], M. Doblaré[b], G.A. Holzapfel[d,e,*]

[a]Institute for Structural Analysis, Computational Biomechanics, Graz University of Technology, Austria
[b]Group of Structural Mechanics and Materials Modeling, Aragon Institute of Engineering Research (I3A), University of Zaragoza, Spain
[c]Medical Information Technology, Carinthia Tech Institute, University of Applied Sciences, Klagenfurt, Austria
[d]Department of Solid Mechanics, School of Engineering Sciences, Royal Institute of Technology (KTH), Stockholm, Sweden
[e]Institute for Biomechanics, Graz University of Technology, Austria

Received 30 December 2006; received in revised form 6 February 2007; accepted 6 February 2007

Abstract

A micromechanically based constitutive model for fibrous tissues is presented. The model considers the randomly crimped morphology of individual collagen fibers, a morphology typically seen in photomicrographs of tissue samples. It describes the relationship between the fiber endpoints and its arc-length in terms of a measurable quantity, which can be estimated from image data. The collective mechanical behavior of collagen fibers is presented in terms of an explicit expression for the strain-energy function, where a fiber-specific random variable is approximated by a Beta distribution. The model-related stress and elasticity tensors are provided. Two representative numerical examples are analyzed with the aim of demonstrating the peculiar mechanism of the constitutive model and quantifying the effect of parameter changes on the mechanical response. In particular, a fibrous tissue, assumed to be (nearly) incompressible, is subject to a uniaxial extension along the fiber direction, and, separately, to pure shear. It is shown that the fiber crimp model can reproduce several of the expected characteristics of fibrous tissues.

Keywords: Collagen fiber; Constitutive model; Fiber crimp; Micromechanics; Statistical distribution

1. Introduction

The central role of collagen as the major structural protein of mammalian tissue, comprising approximately one-third of the total protein in mammalian organisms, has motivated a significant effort towards determining its mechanical properties at all levels, ranging from single monomers [1,2] and long-chain polymers [3,4] to a structural element within a (macroscopic) biological tissue [5–8].

On the basis of the mechanical properties, a number of constitutive models have been developed in the past in attempts to describe the experimental data. While at the microscopic level, chain models such as the (Kratky–Porod) worm-like model are popular [9–11], at the macroscopic level the continuum theory of finite elastic deformations of solids reinforced with fibers is frequently the constitutive theory of choice. The basic ideas of the theory are contained in [12], with further developments on strongly anisotropic solids in [13], and applications to model, e.g., arterial walls in [14,15]; see also the recent volume [16]. In such macroscopic models the collagen fibers are assumed to be continuously arranged in the matrix material, as utilized in [17], and the characteristic non-linear stiffening is best represented by means of an exponential function. Effective alternatives are based on limiting chain models, see, for example, [18], and references therein.

The pioneering work by Lanir [19,20] on the mechanics of fibrous (connective) tissues as a consequence of its microstructure has influenced much of the works on microstructural constitutive models. Essentially, the works [19,20] postulate that the fibers are crimped and that they have different lengths so that for a given macroscopic deformation in the material each individual fiber is stretched differently. There is, thus, a distribution in either the stretch of the fibers or their lengths.

* Corresponding author. Institute for Biomechanics, Center for Biomedical Engineering, Graz University of Technology, Kronesgasse 5, 8010 Graz, Austria.

E-mail address: gh@biomech.tu-graz.ac.at (G.A. Holzapfel).

0020-7462/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijnonlinmec.2007.02.002

This idea has also been adopted subsequently by means of constitutive models to describe the mechanical response of, e.g., arterial walls ([21] with ideas from [22,14]) or tendons and ligaments [23], just to name a few. All these constitutive models, however, assume unbounded statistical distributions for the fiber length (or stretch), which is a bounded quantity. In addition, in these models no attempt has been made to correlate the fiber morphology (crimp) with the associated mechanical response in the form of stress–stretch relationships. It was Lanir who considered the possibility that the stretch could be non-uniform due to crimping, with a generic distribution along the fiber axis, which he assumed to be Gaussian. Recently, Freed and Doehring [24] have proposed a model where crimped fibrils in a fascicle are approximated as a helical spring. Thereby, the collagen fiber waveforms have a pre-defined arrangement; no statistical distribution is used. In different works, such as [25–27], the distribution of the fiber orientations has been addressed; however, therein, the mechanical properties of the collagen fibers within the tissue were considered to be independent of the degree of crimping.

In this paper a new constitutive model for the macroscopic behavior of fibrous tissues is presented. It takes the randomly crimped morphology of the individual collagen fibers into account. In Section 2 a statistical description of the fiber crimp is developed, which is used in Section 3 to model the collective behavior of fibers. In Section 4 the mechanical behavior of a fibrous tissue, assumed to be (nearly) incompressible, is analyzed in detail. The tissue is subject to a stretch-controlled uniaxial extension along the fiber direction, and, separately, to pure shear. In particular, the effect of the different model parameters on the mechanical response is studied. The final section contains a brief discussion together with a description of some limitations of the proposed constitutive model.

2. Statistical and constitutive description of a single collagen fiber

In unloaded tissue samples collagen fibers show a wavy structure. In this section we develop a model that incorporates the random crimp of collagen fibers to be characterized.

2.1. Random crimp of a single fiber

We start by considering a set of randomly generated data in an interval of length $L_0 + w$ on the X-axis such that at any point x within that interval the associated coordinates y and z are independent and normally distributed random variables with zero mean. Under this condition the data generated can be regarded as white Gaussian noise, and characterized by the variance σ^2. In the following it is assumed that the variances in the Y- and Z-directions are equal, in other words the fiber undulates with equal characteristics in all directions orthogonal to the X-axis.

The randomness of the data generated may be larger than that of an actual fiber. By applying a smoothing function or filter h,

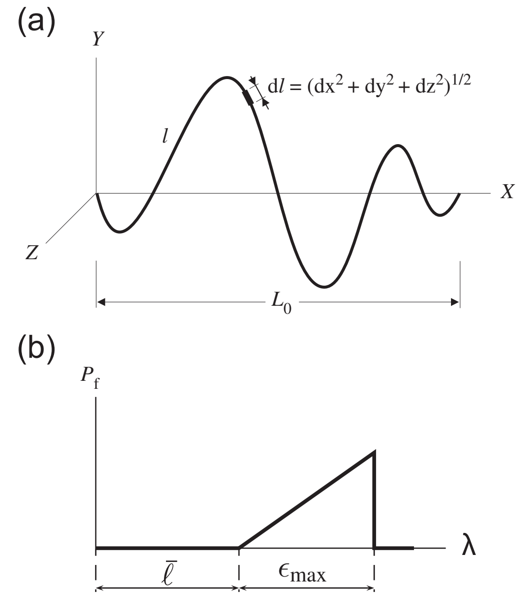

Fig. 1. (a) Schematic representation of a single fiber; (b) stress–stretch behavior of a single fiber (see Eq. (6)).

which averages the coordinates of the points in a neighborhood $[-w/2, w/2]$ of each point, a derived set of data in the interval $[0, L_0]$ is obtained. It is implicitly assumed that h and its first derivative has compact support in $[-w/2, w/2]$. The resulting data are also random and normally distributed with zero mean since the filtering operation does not affect the Gaussian nature of the distribution. As a consequence, the variance of the new random variable is unequivocally related to that of the white Gaussian noise through the filter.

The infinitesimal arc-length $\mathrm{d}l$ of the fiber is then (see Fig. 1(a))

$$\mathrm{d}l = (\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2)^{1/2}, \tag{1}$$

where $\mathrm{d}y$ and $\mathrm{d}z$ are related to $\mathrm{d}x$ through the derivative of the filter. Thus, we can write $d = \mathrm{d}y/\mathrm{d}x = \mathrm{d}z/\mathrm{d}x$, where d is a zero mean, normally distributed random variable whose variance σ_d^2 can be directly related to σ^2. Therefore, it follows that

$$\mathrm{d}l = \sqrt{2d^2 + 1}\,\mathrm{d}x = \ell\,\mathrm{d}x, \tag{2}$$

where ℓ is a random variable, which is neither zero mean nor normally distributed, and whose probability density function, subsequently abbreviated as $\mathscr{P}$, is (see Appendix A.1)

$$\mathscr{P} = \frac{\ell}{\sigma_d^2} \exp\left(-\frac{\ell^2 - 1}{2\sigma_d^2}\right), \quad \ell \geqslant 1. \tag{3}$$

Relation (2) describes the arc-length at infinitesimal scale within a single fiber. Our interest is, however, the establishment of (2) at the fiber level, i.e.

$$l = \int_{x=0}^{L_0} \ell\,\mathrm{d}x \approx \bar{\ell} L_0, \tag{4}$$