International Journal of Mechanical Sciences 52 (2010) 1437–1457

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Non-linear vibrations and instabilities of orthotropic cylindrical shells with internal flowing fluid

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ARTICLE INFO

Article history:
Received 10 September 2009
Received in revised form
13 March 2010
Accepted 25 March 2010
Available online 31 March 2010

Keywords:
Cylindrical shells
Fluid–structure interaction
Non-linear vibrations
Orthotropic material

ABSTRACT

In this work, Donnell's non-linear shallow shell equations are used to study the dynamic instability of perfect simply supported orthotropic cylindrical shells with internal flowing fluid and subjected to either a compressive axial static pre-load plus a harmonic axial load or a harmonic lateral pressure. The fluid is assumed to be non-viscous and incompressible and the flow, isentropic and irrotational. An expansion with eight degrees of freedom, containing the fundamental, companion, gyroscopic, and four axi-symmetric modes is used to describe the lateral displacement of the shell. The Galerkin method is used to obtain the non-linear equations of motion which are solved by the Runge–Kutta method. A detailed parametric analysis clarifies the influence of the orthotropic material properties on the non-linear buckling and vibration characteristics of the shell. Numerical methods are used to identify the effect of the fluid flow and applied loads control parameters on the bifurcations and stability of the shell motions.

1. Introduction

Cylindrical shells are widely used structures in several engineering areas for conveying fluid, being one of the most common shell geometries in industrial applications. The buckling and vibration analysis of cylindrical shells under various loading conditions has thus become an important research area in applied mechanics. Also, the analysis of fluid–shell interaction has been a topic of continuous interest in the last decades. However, most of these investigations are concerned with the analysis of elastic isotropic shells in contact with internal and external quiescent or flowing fluids.

A reduced number of these works has as object of investigation, the behavior of orthotropic shells. Among the previous studies related to the present investigation, we can mention the work of Jain [1] who studies the free vibrations of orthotropic cylindrical shells partially or completely filled with an incompressible, non-viscous fluid; Warburton and Soni [2] study the resonant response of orthotropic cylindrical shells, while Bradford and Dong [3] investigate the lateral vibrations of orthotropic cylinders under initial stress.

Ip et al. [4], using Love's first-approximation shell theory, describe a procedure to model the free vibration responses of fiber-reinforced composite cylindrical shells in a free–free configuration. A set of linear equations of motion is derived using the Rayleigh–Ritz method where the shell vibration mode shapes are described by characteristic beam functions. With that model, inextensional Rayleigh and Love modes can be identified having frequencies close to each other. The contributions to the strain energy due to various elastic properties are also investigated. They show that, when increasing the shell thickness, the circumferential modulus provides a major portion of the flexural energy of the vibrating structure, while the longitudinal and in-plane shear moduli contribute mostly to the stretching energy; reducing the shell thickness results in a substantial increase in the ratio of the energies associated with the longitudinal and shear moduli.

Chen et al. [5] study the free vibrations of orthotropic cylindrical shells based on the three-dimensional elasticity theory considering the effect of internal fluid. They obtain the frequency equation of non-axisymmetric free vibration modes of an orthotropic fluid-filled cylindrical shell with arbitrary constant thickness and compare their results with those based on shell theory, and Chen and Ding [6] analyze the free vibrations of fluid-filled transversely isotropic cylindrical shells. A shell is called transversely isotropic if the in-plane properties are different from those in the perpendicular direction.

Using the Sanders–Koiter non-linear shell theory, Selmane and Lakis [7] study the influence of non-linearities associated with the shell wall and with the fluid flow on the dynamics of elastic thin orthotropic cylindrical shells. They consider non-uniform open

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0020-7403/$ - see front matter © 2010 Published by Elsevier Ltd.
doi:10.1016/j.ijmecsci.2010.03.016

submerged cylindrical shells subjected simultaneously to an internal and external fluid flow by using a hybrid finite element method. Results show the influence of non-linearities on the free vibrations of totally submerged open or closed cylindrical shells; both softening and hardening behavior of the shell are observed.

Lakis et al. [8] present a hybrid method to predict the influence of geometric non-linearities on the natural frequencies of an empty laminated orthotropic cylindrical shell. The Sanders–Koiter non-linear strain–displacement relations are used to formulate the shell equations in combination with the finite element method. Results show the influence of axial and circumferential half-waves on the non-linear frequencies of the shell. The analyzed shells display a hardening behavior.

Mao and Williams [9], using a non-linear theory for non-shallow shells, analyze the parametric resonance of orthotropic circular cylindrical shells under harmonically varying axial compression. In the analysis, the transverse shear deformation is taken into account by considering a first-order shell theory. Numerical results show the dependence of the post-critical behavior on the properties of the material, geometry and excitation parameters.

Li and Chen [10], using Flügge's linear shell theory, study the dynamic response of orthotropic circular cylindrical shells subjected to external hydrostatic pressure. To analyze the dynamic response of the shell, the normal mode theory is used. They investigate in detail the effect of shell parameters, external hydrostatic pressure and material properties on the dynamic behavior of the shell.

Recently, Mallon [11] analyzes experimentally and numerically a base-excited thin orthotropic cylindrical shell with a top mass. The shell is made of Poly-ethylene Terephthalate and the orthotropy is due to the fabrication process [12]. Daneshjou et al. [13] conduct an analytical study to understand the characteristic of sound transmission through an orthotropic cylindrical shell with subsonic external flow.

Apart from naturally orthotropic materials, several shell problems can be effectively analyzed using an orthotropic shell theory. For example, the stability and vibration analysis of densely ring- and string-stiffened cylindrical shells, such as those used in aero-space engineering structures, are usually accomplished by replacing the stiffened structure by an equivalent orthotropic continuum and the effect of stiffeners is averaged or ‘‘smeared out’’ over the shell [14]. Through this method, when the wavelength of vibration is much larger than the distance between stiffeners, very accurate results are achieved. Shen [15] studies the post-buckling analysis of stiffened laminated cylindrical under combined external liquid pressure and axial compression, the ‘smeared stiffener’ approach is adopted for the stiffeners. The analysis uses a singular perturbation technique to determine the interactive buckling loads and post-buckling equilibrium paths. Torkamani et al. [16] investigate experimentally and numerically the free vibrations of orthogonally stiffened cylindrical shells using the similitude theory. The Donnell-type non-linear strain–displacement relations along with the smearing theory are used to model the structure.

The analysis of corrugated shells (plates) can also be analyzed as thin, equivalent orthotropic shells of uniform thickness. Briassoulis [17] derived equivalent orthotropic properties of corrugated sheets and reviewed some analytical expressions for the equivalent rigidities of orthotropic thin shells given in the literature. Using this approach, Wang et al. [18] study the non-linear free vibrations of corrugated circular plates, obtaining the analytical solutions for the amplitude–frequency relationship through a perturbation method. Liu and Li [19] study the non-linear bending and free vibration for corrugated circular plate via Galerkin's and a modified iteration method. Larbi [20] studies the buckling of corrugated tin cans under uniform external pressure modeled as thin-walled orthotropic cylindrical shells. Finally, many problems in biology can also be investigated using the orthotropic shell theory [21].

In this work, the non-linear vibrations of a simply supported fluid-filled orthotropic cylindrical shell subjected to axial time-dependent loads and lateral harmonic pressure are analyzed. To model the shell, the Donnell non-linear shallow shell theory without considering the effect of shear deformation is used. The fluid is assumed to be incompressible and inviscid and the flow to be isentropic and irrotational. The incompressibility hypothesis is true for liquid-filled shells vibrating in the low-frequency range. A model with eight degrees of freedom, satisfying the relevant boundary and continuity conditions, and containing the fundamental, companion, gyroscopic, and four axi-symmetric modes, is used to describe the lateral displacements of the shell and the Galerkin method is applied to derive a set of coupled non-linear ordinary differential equations of motion which are, in turn, solved by the Runge–Kutta method. The statically pre-loaded shell is considered to be initially at rest, in a position corresponding to a pre-buckling configuration. The results clarify the marked influence of the orthotropic material properties, fluid flow and load parameters on the dynamic stability boundaries and bifurcations. To the authors' best knowledge, no such detailed analysis of the influence of material orthotropy on the non-linear buckling and dynamic behavior of cylindrical shells with fluid flow can be found in literature. The basic theory backing the present investigation can be found in Païdoussis [22] and Amabili [23].

2. Mathematical formulation

2.1. Shell equations

Consider a thin-walled simply supported cylindrical shell with radius R, thickness h, length L, containing an internal flowing fluid and subjected to either a harmonic lateral pressure or a time-dependent axial load. The middle surface of the shell is defined as the reference surface. The axial, circumferential and radial coordinates are denoted by x, $y=R\theta$, and z, respectively, and the corresponding displacements of the shell middle surface are denoted by u, v, and w, as shown in Fig. 1. It is assumed that the local coordinate system, which determines the principal axes of material orthotropy, coincides with the global cylindrical coordinates. The shell is made of an elastic orthotropic material with Young's moduli E_{xx} and $E_{\theta\theta}$ in the axial and circumferential directions, respectively, shear modulus $G_{x\theta}$, Poisson coefficients ν_{xx} and $\nu_{\theta\theta}$, and mass density ρ_s. In this work the mathematical formulation follows that previously presented in Refs. [24–27].

For an orthotropic material, obeying the generalized Hooke's law, the stress resultant-strain relations are given by

$$\begin{Bmatrix} N_{xx} \\ N_{\theta\theta} \\ N_{x\theta} \\ M_{xx} \\ M_{x\theta} \\ M_{\theta\theta} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & 0 & 0 & 0 \\ A_{21} & A_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{11} & D_{12} & 0 \\ 0 & 0 & 0 & D_{21} & D_{22} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{33} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{\theta\theta} \\ \varepsilon_{x\theta} \\ \kappa_{xx} \\ \kappa_{\theta\theta} \\ \kappa_{x\theta} \end{Bmatrix} \tag{1}$$

where N_{xx}, $N_{\theta\theta}$, and $N_{x\theta}$ are the in-plane normal and shearing force intensities per unit length along the edge of a shell element; M_{xx}, $M_{x\theta}$, and $M_{\theta\theta}$ the bending and twisting moment resultants; ε_{xx} and $\varepsilon_{\theta\theta}$ the extensional strain in the axial and circumferential directions and $\varepsilon_{x\theta}$ the shearing strain components at a point on the shell middle surface; κ_{xx} and $\kappa_{\theta\theta}$ the curvature changes and