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Pulsatile flow of Herschel–Bulkley fluid through stenosed arteries—A mathematical model

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Abstract

In this paper, the pulsatile flow of blood through stenosed artery is studied. The effects of pulsatility, stenosis and non-Newtonian behavior of blood, assuming the blood to be represented by Herschel–Bulkley fluid, are simultaneously considered. A perturbation method is used to analyze the flow assuming the thickness of plug core region to be non-uniform changing with axial distance. An expression for the variation of plug core radius with time and axial distance is obtained. The variation of pressure gradient with steady flow rate is given. Also the variation of wall shear stress distribution as well as resistance to flow with axial distance for different values of time and for different values of yield stress is given and the results analyzed.

Keywords: Blood rheology; Pulsatile flow; Herschel–Bulkley fluid; Stenosed arteries; Wormersly frequency parameter

1. Introduction

The study of blood flow through a stenosed tube plays an important role in the fundamental understanding, diagnosis and treatment of many cardiovascular diseases. For low shear rate flow in narrow tubes, particularly in diseased state blood may be represented by a non-Newtonian fluid and also the actual blood flow is distinctly pulsatile. Among the various arterial diseases the development of arteriosclerosis in blood vessels is quite common which may be attributed to accumulation of lipids in the arterial wall or pathological changes in the tissue structure [1]. Arteries are narrowed by the development of atherosclerotic plaques that protrude into the lumen, resulting in stenosed arteries. As an obstruction develop in an artery, one of the most serious consequences is the increased resistance and the associated reduction of blood flow to the particular vascular bed supplied by the artery. Also the continual flow of blood may lead to shearing of the superficial layer of the plaques, parts of which may be deposited in some other blood vessel forming thrombus. Thus the presence of a stenosis can lead to serious circulatory disorder.

Numerous investigators have cited hydrodynamic factors playing an important role in the formation of stenosis [2,3] and hence the mathematical modeling of blood flow through a stenosed tube is very important. Many authors have dealt with this problem treating blood as a Newtonian fluid and assuming the flow to be steady [4–7]. The Newtonian behavior may be true in larger arteries, but blood, being a suspension of cells in plasma, exhibits non-Newtonian behavior at low shear rates in small arteries [8–12]. Shukla et al. [13] and Shukla et al. [14] and Chaturani and Ponnalagar Samy [15] have studied the non-Newtonian behavior for steady flows in stenosed tubes.

It is well known that blood consists of multiple constituents namely red blood cells (RBCs), white blood cells, platelets, etc., suspended in a medium (plasma) of proteins and water. The plasma is a Newtonian fluid. Chien et al. [16,17] have related the shear-thinning nature of blood to the tendency of RBC-rouleau aggregates (which form at low shear rates) to disaggregate upon the application of shear. Upon increasing the shear rate, the RBCs become ‘fluid like’ and lose the ability to store

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elastic energy [18]; they align themselves with the flow field and tend to slide upon plasma layers formed in between [19]. Among the researchers, Thurston [20] was the first to recognize the viscoelastic nature of blood and that the viscoelastic nature of blood is less prominent with increasing shear rate [21,22]. As the seminal contribution to the study of shear thinning viscoelastic nature of blood, Thurston [22] proposed an extended Maxwell model applicable to one-dimensional flow situations. Thurston [22] has also observed that there exist a critical shear rate beyond which the assumptions of linear viscoelasiticity and Newtonian behavior of blood cease to hold and related the non-linear behavior to the microstructural changes that occur in blood with increasing shear rate [23,24]. Cho and Kensey [25] have numerically studied the effects of non-Newtonian viscosity of blood on flows in a diseased arterial blood vessel using a finite element method. The most recent Oldroyd-B type model of Yeleswarapu et al. [26] is an improvement over earlier proposals like that due to Philips and Deutsch [27]. Anand and Rajagopal [28] have studied extensively a shear-thinning viscoelastic fluid model for blood flow within a thermodynamic framework that takes cognizance of the fact that viscoelastic fluids can remain stress free in several configurations.

It is found that arterial blood flow is highly pulsatile with marked effects on instantaneous velocity distributions and the flow rate varies over a wide range during a flow cycle [7,29]. For Newtonian fluids, the problem of unsteady flow has been studied by quite a few researchers [30–34]. However, a flattened velocity profile has been obtained especially when dealing with pulsatile flow in many experiments suggesting a plug flow in the core region. Therefore, it is appropriate to represent blood as a non-Newtonian fluid which models this velocity profile. Aroesty and Gross [35,36] and Chaturani and Ponnalagar Samy [37] have studied the pulsatile flow of Casson's fluid through stenosed arteries. In this paper we have extended their result for pulsatile flow of Herschel–Bulkley fluid through a stenosed tube, as it is noticed that blood obeys Casson's equation only for moderate shear rate and the Herschel–Bulkley equation represents fairly closely what is occurring in blood [38]. Chaturani et al. [15] have mentioned that for tube diameter 0.095 mm blood behaves like Herschel–Bulkley fluid rather than power law and Bingham fluids. Iida [39] reports "The velocity profile in the arterioles having diameter less than 0.1 mm are generally explained fairly by the two models. However, velocity profiles in the arterioles whose diameters are less than 0.065 mm does not conform to the Casson model but can still be explained by Herschel–Bulkley model. Hence, we felt that it is appropriate to model blood as a Herschel–Bulkley fluid than Casson fluid in this paper.

Tu and Deville [40] have solved numerically the pulsatile flow of Herschel–Bulkley fluid through stenosed arteries using a Galerkin finite element method. In this paper the effects of pulsatility, stenosis and non-Newtonian behavior of blood, assuming the blood to be represented by Herschel–Bulkley fluid, are simultaneously considered using an analytical solution. A perturbation method is used to analyze the flow assuming the thickness of plug core region to be non-uniform changing with axial distance. An expression for the variation of plug core radius with time and axial distance is obtained. The variation of pressure gradient with steady flow rate is given. Also the variation of wall shear stress distribution as well as resistance to flow with axial distance for different values of time and for different values of yield stress is given and the results analyzed. The results for Newtonian fluid and power law fluid can be obtained as particular cases from the present model.

2. Mathematical formulation

Consider an axially symmetric, laminar, pulsatile and fully developed flow of blood (assumed to be incompressible) in the $\bar{z}$ direction through a circular artery with a mild stenosis as shown in Fig. 1. We have used cylindrical polar coordinates $(\bar{r}, \bar{\phi}, \bar{z})$ whose origin is located on the vessel (stenosed artery) axis. The walls of the stenosed artery are assumed to be rigid.

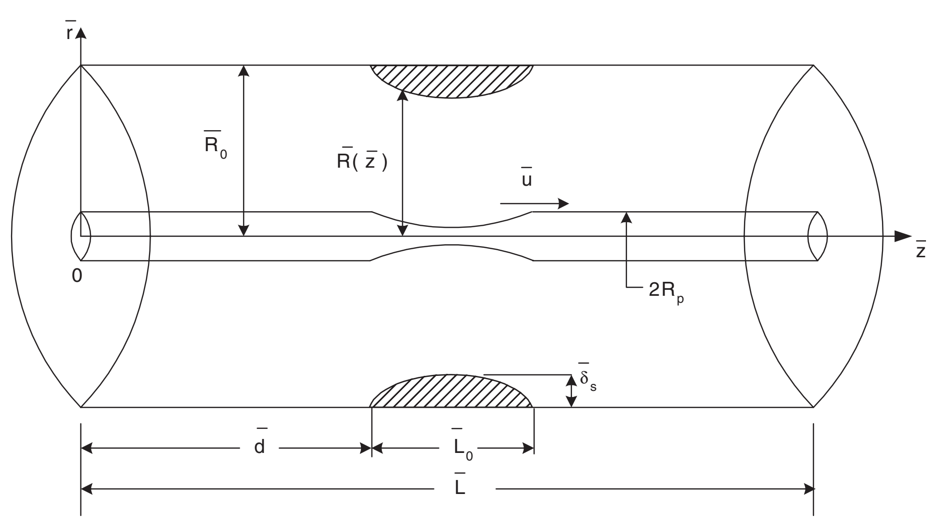

Fig. 1. Geometry of stenosed artery.