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Free vibration of laminated composite plates using two variable refined plate theory

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A B S T R A C T

Free vibration of laminated composite plates using two variable refined plate theory is presented in this paper. The theory accounts for parabolic distribution of the transverse shear strains through the plate thickness, and satisfies the zero traction boundary conditions on the surfaces of the plate without using shear correction factors. Equations of motion are derived from the Hamilton's principle. The Navier technique is employed to obtain the closed-form solutions of antisymmetric cross-ply and angle-ply laminates. Numerical results obtained using present theory are compared with three-dimensional elasticity solutions and those computed using the first-order and the other higher-order theories. It can be concluded that the proposed theory is not only accurate but also efficient in predicting the natural frequencies of laminated composite plates.

1. Introduction

Laminated composite plates are widely used in the aerospace, automotive, marine and other structural applications because of advantageous features such as high ratio of stiffness and strength to weight and low maintenance cost. In company with the increase in the application of laminates in engineering structures, a variety of laminated theories have been developed. The classical laminated plate theory (CLPT), which neglects the transverse normal and shear stresses, provides reasonable results for thin laminates. However, it underpredicts deflections and overpredicts frequencies as well as buckling loads with moderately thick laminates [1]. In order to overcome the limitations of CLPT, the shear deformation theories accounted for the effect of transverse shear deformation have been recommended. The first-order shear deformation theory (FSDT) assumes linear variation of in-plane displacements through the thickness. Many studies of the free vibration of laminates have been carried out using FSDT [2–4]. Since FSDT violates equilibrium conditions at the top and bottom faces of the plate, shear correction factors are required to correct the unrealistic variation of the shear strain/stress through the thickness. The value of shear correction factor depends not only on the lamination and geometric parameters, but also on the loading and boundary conditions. To avoid the use of shear correction factors, the higher-order shear deformation theories (HSDT) based on power series expansion of displacements with respect to the thickness coordinate have been developed. The HSDT has been widely used to investigate the free vibration of laminated plates [5–10]. A review of various shear deformation theories for the analysis of laminated composite plates is available in Refs. [11–13]. Recently, a two variable refined plate theory (RPT) was first developed for isotropic plates by Shimpi [14], and was extended to orthotropic plates by Shimpi and Patel [15,16] and Kim et al. [17]. The most interesting feature of this theory is that it does not require shear correction factor, and has strong similarities with the CLPT in some aspects such as governing equation, boundary conditions and moment expressions. Kim et al. [18] has developed this theory for the laminated composite plates. The accuracy of this theory has been demonstrated for static bending and buckling analyses of laminates by Kim et al. [18], therefore, it seems to be important to extend this theory to the free vibration analysis of laminates.

The purpose of this paper is to extend the RPT developed by Kim et al. [18] to the free vibration of laminated composite plates. Equations of motion are derived from the Hamilton's principle. The closed-form solutions for simply supported antisymmetric cross-ply and angle-ply laminates are obtained using Navier solution. The effects of parameters such as the aspect ratio, thickness ratio, modulus ratio and number of layers on the natural frequencies of the laminates are investigated. Numerical examples are presented to illustrate the accuracy and efficiency of the present theory in predicting the natural frequencies of laminates by comparing the predictions with those computed using various theories and the exact solutions of three-dimensional elasticity theory.

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2. RPT for laminated composite plates

2.1. Basic assumptions

Consider a rectangular plate of total thickness h composed of n orthotropic layers with the coordinate system as shown in Fig. 1. Assumptions of the RPT are as follows:

(1) The displacements are small in comparison with the plate thickness and, therefore, strains involved are infinitesimal.
(2) The transverse displacement W includes three components of extension w_a, bending w_b, and shear w_s. Both these components are functions of coordinates x, y, and time t only.

$$W(x,y,z,t)=w_a(x,y,t)+w_b(x,y,t)+w_s(x,y,t) \tag{1}$$

(3) The transverse normal stress σ_z is negligible in comparison with in-plane stresses σ_x and σ_y.
(4) The displacements u in x-direction and v in y-direction consist of extension, bending, and shear components:

$$U=u+u_b+u_s \quad \text{and} \quad V=v+v_b+v_s \tag{2}$$

- The bending components u_b and v_b are assumed to be similar, respectively, to the displacements given by the classical plate theory. Therefore, the expression for u_b and v_b can be given as

$$u_b=-z\frac{\partial w_b}{\partial x} \quad \text{and} \quad v_b=-z\frac{\partial w_b}{\partial y} \tag{3}$$

- The shear components u_s and v_s give rise, in conjunction with w_s, to the parabolic variations of shear strains γ_{xz}, γ_{yz} and hence to shear stresses σ_{xz}, σ_{yz} through the thickness of the plate in such a way that shear stresses σ_{xz}, σ_{yz} are zero at the top and bottom faces of the plate. Consequently, the expression for u_s and v_s can be given as

$$u_s=\left[\frac{1}{4}z-\frac{5}{3}z\left(\frac{z}{h}\right)^2\right]\frac{\partial w_s}{\partial x} \quad \text{and} \quad v_s=\left[\frac{1}{4}z-\frac{5}{3}z\left(\frac{z}{h}\right)^2\right]\frac{\partial w_s}{\partial y}. \tag{4}$$

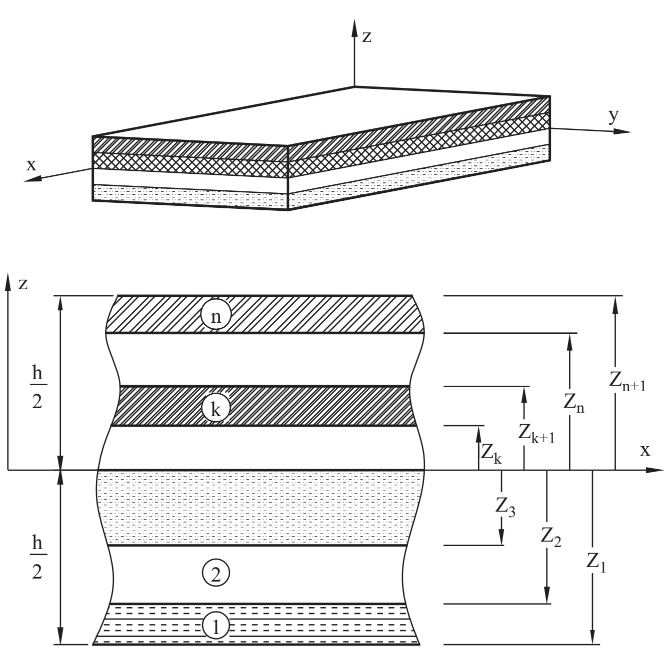

Fig. 1. Coordinate system and layer numbering used for a typical laminated plate.

2.2. Kinematics

Based on the assumptions made in the preceding section, the displacement field can be obtained using Eqs. (1)–(4) as

$$\begin{aligned}
U(x,y,z,t)&=u(x,y,t)-z\frac{\partial w_b}{\partial x}+z\left[\frac{1}{4}-\frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial w_s}{\partial x}\\
V(x,y,z,t)&=v(x,y,t)-z\frac{\partial w_b}{\partial y}+z\left[\frac{1}{4}-\frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial w_s}{\partial y}\\
W(x,y,z,t)&=w_a(x,y,t)+w_b(x,y,t)+w_s(x,y,t)
\end{aligned} \tag{5}$$

The extension component $w_a(x,y,t)$ of transverse displacement is negligibly small in most cases. It can be neglected for a simpler version of the present theory, and then the displacement field may be expressed as

$$\begin{aligned}
U(x,y,z,t)&=u(x,y,t)-z\frac{\partial w_b}{\partial x}+z\left[\frac{1}{4}-\frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial w_s}{\partial x}\\
V(x,y,z,t)&=v(x,y,t)-z\frac{\partial w_b}{\partial y}+z\left[\frac{1}{4}-\frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial w_s}{\partial y}\\
W(x,y,z,t)&=w_b(x,y,t)+w_s(x,y,t)
\end{aligned} \tag{6}$$

The above-mentioned two displacement models are referred as RPT1 and RPT2 for the simpler and full versions, respectively, in all figures and tables. The strains associated with the displacements in Eq. (5) are

$$\begin{aligned}
\varepsilon_x&=\varepsilon_x^0+z\kappa_x^b+f\kappa_x^s\\
\varepsilon_y&=\varepsilon_y^0+z\kappa_y^b+f\kappa_y^s\\
\gamma_{xy}&=\gamma_{xy}^0+z\kappa_{xy}^b+f\kappa_{xy}^s\\
\gamma_{yz}&=\gamma_{yz}^a+g\gamma_{yz}^s\\
\gamma_{xz}&=\gamma_{xz}^a+g\gamma_{xz}^s\\
\varepsilon_z&=0
\end{aligned} \tag{7}$$

where

$$\begin{aligned}
&\varepsilon_x^0=\frac{\partial u}{\partial x}, \quad \kappa_x^b=-\frac{\partial^2 w_b}{\partial x^2}, \quad \kappa_x^s=-\frac{\partial^2 w_s}{\partial x^2}\\
&\varepsilon_y^0=\frac{\partial v}{\partial y}, \quad \kappa_y^b=-\frac{\partial^2 w_b}{\partial y^2}, \quad \kappa_y^s=-\frac{\partial^2 w_s}{\partial y^2}\\
&\gamma_{xy}^0=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}, \quad \kappa_{xy}^b=-2\frac{\partial^2 w_b}{\partial x\partial y}, \quad \kappa_{xy}^s=-2\frac{\partial^2 w_s}{\partial x\partial y}\\
&\gamma_{yz}^a=\frac{\partial w_a}{\partial y}, \quad \gamma_{yz}^s=\frac{\partial w_s}{\partial y}\\
&\gamma_{xz}^a=\frac{\partial w_a}{\partial x}, \quad \gamma_{xz}^s=\frac{\partial w_s}{\partial x}\\
&f=-\frac{1}{4}z+\frac{5}{3}z\left(\frac{z}{h}\right)^2, \quad g=\frac{5}{4}-5\left(\frac{z}{h}\right)^2
\end{aligned} \tag{8}$$

2.3. Constitutive equations

Under the assumption that each layer possesses a plane of elastic symmetry parallel to the x-y plane, the constitutive equations for a layer can be written as

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_{xy}\\ \sigma_{yz}\\ \sigma_{xz}\end{Bmatrix}=\begin{bmatrix}Q_{11} & Q_{12} & 0 & 0 & 0\\ Q_{12} & Q_{22} & 0 & 0 & 0\\ 0 & 0 & Q_{66} & 0 & 0\\ 0 & 0 & 0 & Q_{44} & 0\\ 0 & 0 & 0 & 0 & Q_{55}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} \tag{9}$$

where Q_{ij} are the plane stress-reduced stiffnesses defined in terms of the engineering constants in the material axes of the