International Journal of Mechanical Sciences 52 (2010) 75–88

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Nonlinear in-plane elastic buckling of shallow circular arches under uniform radial and thermal loading

Yong-Lin Pi *, Mark Andrew Bradford

Centre for Infrastructure Engineering and Safety, School of Civil and Environmental Engineering, The University of New South Wales, Sydney, NSW 2052, Australia

ARTICLE INFO

Article history:
Received 4 November 2008
Received in revised form
25 August 2009
Accepted 29 October 2009
Available online 6 November 2009

Keywords:
Bifurcation
Buckling
Limit instability
Non-linear
Shallow arches
Thermoelastic

ABSTRACT

This paper presents a nonlinear in-plane elastic buckling analysis of circular shallow arches that are subjected both to a uniform temperature field and to a uniform radial load field. A virtual work method is used to establish nonlinear equilibrium equations and buckling equilibrium equations, and analytical solutions for the limit instability and bifurcation buckling loads are obtained. It is found that the temperature influences the limit instability, bifurcation buckling and postbuckling behaviour of shallow arches significantly. The limit instability and bifurcation buckling loads increase with an increase of the temperature. A maximum temperature is shown to exist for the occurrence of bifurcation buckling of shallow arches, and when the temperature is higher than this value, bifurcation buckling of an arch is not possible.

An arch geometric parameter is introduced to define switches between the limit instability and bifurcation buckling modes, and between buckling and no buckling. Formulae and methods for the calculation of the limiting values of the arch geometric parameter are developed. It is also found that the limiting values of the arch geometric parameter decrease with an increase of the temperature.

1. Introduction

The circular arch shown in Fig. 1 is a relatively simple but very useful structural form and has been used in civil engineering construction widely. An arch has an important structural response associated with elastic buckling and it may buckle in a limit instability mode (Fig. 2(a)) or in a bifurcation mode from the primary equilibrium path to a secondary equilibrium path (Fig. 2(b)). In-plane elastic buckling of shallow arches has been studied by a number of authors [1–15]. In addition to external load fields, arches may also be subjected to temperature fields. However, most studies on in-plane elastic buckling of arches [1–15] have not considered the thermal effects of temperature and studies of in-plane elastic buckling of arches under thermal effects appear to be reported in the open literature rarely.

Elastic buckling of straight members under thermal effects have been studied [16–20] as summarized in [18,19]. However, the thermoelastic behaviour of an arch is quite different from that of straight members. Michida et al. [21] studied the snap-through action of arch springs and movable bimetallic springs of thermo-switches. It was found because one end of the arch is movable and a rise of temperature can cause snap-through. Bradford [22] studied the thermal buckling of axial-elastically restrained shallow circular arches under a uniform temperature field without external loading. The arch ends were axially movable, but the movements were retrained by axial elastic restraints. It was reported that an arch cannot buckle in a bifurcation mode under the uniform temperature field and that symmetric buckling is possible only when the arch is very flat. Xi and Li [23] investigated the nonlinear stability of deep fixed functional graded material arches under mechanical and thermal loading, with linear elastic materials being a special case. Bradford and Pi [24] developed a curved-beam element for thermoelastic buckling analysis of elastic supported arches, while Pi and Bradford [25] recently performed a systematic investigation of pin-ended and fixed arches, which are subjected to a uniform temperature field. It was found that the uniform temperature leads the arch to displace axially and radially, and produces axial compressive forces and bending moments in the arch. Typical variations of the dimensionless central radial displacement v_c/L of pin-ended arches produced by a uniform temperature field $\Delta T = (T - 20^\circ) = 80\,^\circ C$ (i.e. the ambient temperature is assumed to be $20\,^\circ C$) with the included angle 2Θ are shown in Fig. 3 for arches with a span $L = 6$ m and $r_x = 108$ mm, where r_x is the radius of gyration about the major principal axis of the cross-section. It can be seen that the displacements are upward, i.e. in the convex direction of the arch and are substantial for shallow arches. Typical variations of the central axial compressive forces N_c and bending moment M_c of pin-ended

* Corresponding author.
E-mail address: y.pi@unsw.edu.au (Y.-L. Pi).

0020-7403/$ - see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijmecsci.2009.10.011

arches produced by the temperature field $\Delta T=(T-20^{\circ})=80\,^{\circ}$C with the included angle 2Θ are shown in Fig. 4, where A is the area of the cross-section, I_x is the second moment of area of the cross-section about its major principal axis, and E is Young's modulus at the temperature T, which is a function of temperature and which, for example, is given for steel empirically by [26]

$$E=\begin{cases} E_{20}\times\left[1.0+\dfrac{T}{2000\ln(T/1100)}\right] & \text{for } 0\,^{\circ}\text{C}<T\le 600\,^{\circ}\text{C},\\ E_{20}\times\dfrac{690(1-T/1000)}{T-53.5} & \text{for } 600\,^{\circ}\text{C}<T\le 1000\,^{\circ}\text{C},\end{cases} \quad (1)$$

where E_{20} is Young's modulus of steel at the ambient temperature $20\,^{\circ}$C. It can be seen from Fig. 4 that the axial compressive forces and bending moments are substantial for shallow arches. The substantial radial displacements, axial compressive force and bending moments due to the uniform temperature field may influence the buckling of shallow arches. Eq. (1) is used for the Young's modulus at the temperature T in the numerical computations throughout this paper.

When an arch is subjected to both a radial uniform load field and a temperature field, the axial compressive force produced by the temperature increases the compressive force produced by the radial load. On the other hand, the displacements and bending moments produced by the temperature are opposite to the displacements and bending moments produced by the radial load and so the increase of temperature reduces the radial displacements and bending moments produced by the radial load. It is not known how these upward radial displacements, the axial compressive forces and the bending moments produced by the uniform temperature field influence the nonlinear buckling and postbuckling behaviour of a shallow arch. Hence, the nonlinear buckling and postbuckling analysis of a shallow arch under thermal effects is needed.

The purposes of this paper are to extend the work of Bradford [22] to an investigation of the nonlinear in-plane elastic buckling and postbuckling of circular shallow pin-ended and fixed arches (Fig. 1) that are subjected both to a radial load uniformly distributed around the arch axis and to a uniform temperature field, to derive analytical closed form solutions for the nonlinear limit instability and bifurcation buckling of an arch and for the limiting arch geometric parameter for switches between the buckling modes and between buckling and no buckling under both uniform radial and thermal loading, and to investigate the effects of temperature on the buckling and postbuckling behaviour of these arches.

2. Nonlinear in-plane equilibrium

2.1. Differential equation of equilibrium

The basic assumptions used in the investigation are:

1. arches are slender, i.e. the dimensions of the cross-section are much smaller than the length of the centroidal axis of the arch S and the initial radius of the circular arch R. Hence, deformations of arches are assumed to satisfy the Euler–Bernoulli hypothesis, i.e. the cross-section remains plane and perpendicular to the arch axis during deformation;
2. the derivative of the temperature T with respect to time t vanishes, i.e. $dT/dt=0$ and the temperature gradient also vanishes, i.e. $\nabla T=0$; and so the temperature field T is constant and uniform;
3. the cross-section is constant and uniform along the arch axis;
4. the coefficient of thermal expansion α is independent of the temperature T;
5. it is customary to perform the thermoelastic analysis of such slender members with the same degree of rigor as that accepted in the theory of elasticity; and

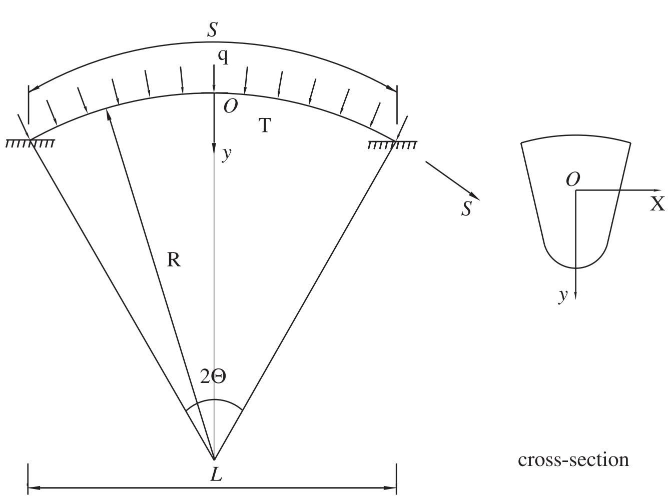

Fig. 1. Arches subjected to a uniform radial load and a uniform temperature field.

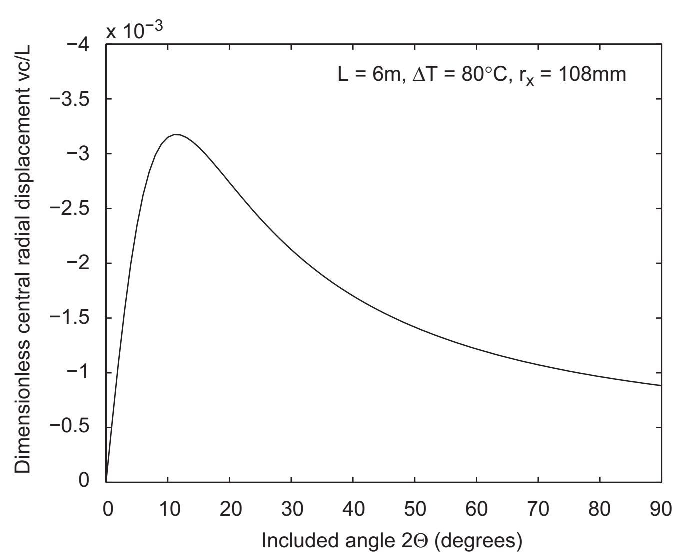

Fig. 3. Central radial displacements of pin-ended arches produced by a uniform temperature field.

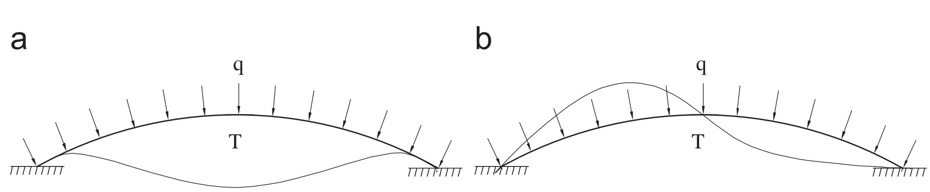

Fig. 2. Buckling modes: (a) symmetric instability and (b) antisymmetric bifurcation buckling.