International Journal of Plasticity 71 (2015) 10–31

Contents lists available at ScienceDirect

International Journal of Plasticity

journal homepage: www.elsevier.com/locate/ijplas

# On the importance of nonlinear elastic effects in shear band modeling

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A R T I C L E I N F O

*Article history:*
Received 16 December 2014
Received in revised form 9 April 2015
Available online 22 April 2015

*Keywords:*
A. Thermomechanical Processes
B. Constitutive Behavior
B. Finite Strain
C. Finite Elements

A B S T R A C T

Shear bands are typically modeled using grade zero hypoelastic formulations based on an objective stress rate, most often the Jaumann rate. Grade zero hypoelastic formulations have been criticized in the literature as unphysical since it can be shown that such a material is not elastic material defined by a hyperelastic potential, and energy is dissipated around a closed loop of deformation. In addition, the Jaumann rate displays a well known oscillation in large shear. However, numerical studies to date comparing hypo and hyperelastic formulations have shown that these issues are not a concern for metal plasticity under roughly monotonic loading, where elastic strains remain small and there is no accumulation of errors from numerous loading cycles. Numerical implementation of Jaumann rate based formulations is relatively easy, and under certain circumstances leads to symmetric tangent moduli. Due to these three factors, use of the Jaumann rate has persisted despite well-known theoretical deficiencies. In this work, we compare hyperelastic and Jaumann rate based hypoelastic formulations for shear band modeling, and it is found that early in the deformation, there are no appreciable differences in the two formulations, which is consistent with previous studies reported in the literature. However, later in the deformation, after localization has occurred, the decrease in elastic moduli brings about the nonlinear elastic effects which causes the hyperelastic formulation to lead to more severe localization and more intense plastic straining than the hypoelastic formulation. This occurs even though the maximum principal elastic stretches do not exceed 1.07 in any of the numerical simulations reported in this work. In addition, the effect of elastic nonlinearity is rate dependent, since the rate dependence of plasticity increases the yield stress with increasing strain rate, which leads to larger elastic strains, and more prominent reduction in elastic moduli. Therefore, for shear band modeling, the still popular grade zero hypoelastic formulation based on the Jaumann rate can not be considered a reasonable approximation to the more sound hyperelastic formulation, and should not be used. In light of these results, we may conclude that in addition to the well known fact that use of the Jaumann rate formulation should be constrained to problems with small elastic strains and without cyclical loading, its use should also be constrained to problems where shear banding and localization is not encountered.



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http://dx.doi.org/10.1016/j.ijplas.2015.04.004

## 1. Introduction

Shear banding is a localization phenomenon where thermal softening leads to narrow zones of intense inelastic deformation (Wright, 2002). While shear bands typically precede ductile fracture, since the thermal softening leads to profound and rapid loss of load carrying capability, this phenomenon is considered a failure mode in its own right (Arriaga et al., 2015; Bai, 1982; Fressengeas and Molinari, 1987; Ling and Belytschko, 2009).

Modeling of shear bands is typically based on a grade zero hypoelastic formulation and the objective Jaumann stress rate, see (Batra and Kim, 1992; Belytschko et al., 1994; Bonnet-Lebouvier et al., 2002; Li et al., 2002; McAuliffe and Waisman, 2014; Nacar et al., 1989; Zhou et al., 1996) among others. Use of this type of formulation in the context of elastoplasticity dates back to (Prager, 1961), but since then has been criticized on theoretical grounds. It can be shown that such a formulation is incompatible with hyperelasticity, meaning that it has not been obtained from a stored energy potential, except under unreasonably restrictive circumstances (Simo and Pister, 1984). In addition, such a material is not elastic in the sense that the net work around a closed loop is non zero (Bernstein, 1960; Kojić and Bathe, 1987). These issues are not unique to the Jaumann rate; a grade zero hypoelastic material based on any objective rate except one will not define an elastic material. The one exception is the logarithmic rate which was developed and analyzed in (Bruhns et al., 1999; Xiao et al., 1997a,b, 1999, 2000; Zhu et al., 2014). This model was shown to be equivalent to the rate form of the hyperelastic model based on the logarithmic strain, and is the only rate among infinitely many possibilities for which this is true.

Several hyperelasticty based plasticity models have been developed using Neo Hookean energy potentials (Simo, 1988; Simo and Hughes, 1998; Simo and Miehe, 1992), and hyperelastic potentials based on logarithmic strain measures (Auricchio and Taylor, 1999; Eterovic and Bathe, 1990; Weber and Anand, 1990). These models define truly elastic materials, and objective stress rates are not needed since objectivity of the elastic relations are automatically satisfied. Despite this, grade zero hypoelastic formulations remain quite popular since for metal plasticity, the assumption of constant elastic moduli is generally appropriate because elastic stretches often remain small. This has been borne out by numerical comparisons of hyper and grade zero hypoelastic formulations for simulating various plasticity problems (Anand, 1979; Brepols et al., 2014), which show only minor differences between the two. Comparisons such as this have been used to justify the continued use of grade zero hypoelasticity for plasticity modeling. In addition, referring specifically to Jaumann rate based formulations, a symmetric tangent modulus can be obtained for numerical computations for certain problems, which provides further motivation for use of this formulation.

However, these studies did not examine problems which lead to severe localization, and while hyperelastic models have been used to simulate localization in granular materials (Andrade and Borja, 2006; Borja, 2002; Borja et al., 2013) as well as rubber − like materials (Triantafyllidis and Aifantis, 1986)and general hyperelastic materials (Leroy and Molinari, 1993), to our knowledge the role of nonlinear elasticity in the formation of shear bands have not been studied. In the numerical results shown below, it is found that while the maximum principal elastic stretches remain below 1.07, the effects of nonlinear elasticity lead to significant differences in observed behavior when compared to the often used Jaumann rate based grade zero hypoelastic model. The reason for the marked differences between the formulations found here, in contrast to the minor differences reported in previous work is that the nonlinear elastic effects due to the hyperelastic material law do not play a significant role until the later stages in the deformation. Nonlinear elasticity tends to increase the extent of the deformation after localization has occurred, since the material is more compliant due to the reduction in elastic moduli. This occurs even though the reduction in moduli is fairly small. In addition, the spurious elastic energy dissipation produced by hypoelastic formulations leads to an unphysical increase in the energy dissipation capacity of the material. Since the Jaumann rate based formulation is the most commonly used for shear band modeling, we have only shown results for this one objective rate. However, with the unique exception of the logarithmic rate, qualitatively the above conclusions would not be affected by the choice of objective rate. Therefore, for shear band modeling, a grade zero hypoelastic formulation with the Jaumann rate can not be considered a reasonable approximation to the more sound hyperelastic formulation, and should not be used. Hyperelastic formulations, represented either in total form, or rate form, should be used instead.

Secondly, since the shear band model involves nonsymmetric thermo mechanical coupling, a symmetric tangent for the global system can not be obtained regardless of the elasticity model chosen. This negates the numerical advantage of using the popular Jaumann rate based formulation. It is shown that this behavior is important for applications where the energy absorption of the material is of interest, since the hyperelastic formulation predicts less dissipation than the hypoelastic formulation. While the model here is for metals with thermal softening shear bands, we expect that similar trends would be observed if a comparison of hyper vs hypoelasticity were undertaken for other shear bands models such as gradient plasticity (Anand et al., 2005, 2012; Gurtin and Anand, 2005).

The role of the nonlinear elastic effect increases with an increase in strain rate, since the rate dependence of plasticity delays yielding for higher strain rates, which leads to larger elastic strains and greater reduction of the elastic moduli. This is of interest in the ductile − brittle failure transition, which is governed by the competition between the formation of shear bands due to thermal softening, and the development of a crack due to large principal stress (Needleman and Tvergaard, 1995, 2000). Hyperelasticity will affect both aspects of the transition since as shown here, the nonlinear elastic softening accelerates the formation of a shear band, and as shown by (Buehler et al., 2003), nonlinear elastic softening decrease the energy flow to the crack tip. Thus, in comparison to a linear elastic solid, an elastic softening material will shear band more readily and fracture less readily. For a material that stiffens elastically, the opposite behavior is expected.