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An explicit, direct simulation of multiaxial finite strain inelastic behavior for polymeric solids

H. Xiao [a], [b], [*]

[a] State Key Laboratory of Advanced Special Steels, Shanghai University, China
[b] Shanghai University of Applied Mathematics and Mechanics, Yanchang Road 149, 200072 Shanghai, China

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ABSTRACT

In contrast with the widely known elastoplastic behavior of hard solids with very small elastic deformations, we study elastoplastic behavior of soft solids dominated by very large elastic deformations. Toward this goal, thermo-coupled rate-independent and rate-dependent elastoplastic J_2-flow models with evolving rubberlike elasticity are established for the first time in a sense of identically meeting the Clausius–Duhem inequality. Novel results are presented for coupling effects in three respects: (i) how finite strain elastic behavior may evolve with development of plastic flow; (ii) how plastic flow may be induced in a process of pure heating; and (iii) how strain rate effects may be characterized to ensure smooth transitions to the rate-independent case. It is demonstrated that complex inelastic deformation features observed in soft solids such as elastomers, including the Mullins effect, the permanent set, the induced anisotropy, the thermal recovery and the rate effect etc., may be derived as direct, natural consequences of the proposed model. In particular, explicit expressions for the constitutive functions incorporated are derivable from the uniaxial data for the purpose of achieving an explicit, exact simulation of the foregoing inelastic features, thus bypassing usual complexities both in choosing suitable forms of constitutive functions and in identifying unknown parameters in an approximate sense.

1. Motivation and introduction

Soft solids, such as biological soft tissues and elastomeric materials, in particular, filled rubbers and crystallizing pure gums, exhibit very complex non-elastic deformation features, such as the Mullins effect associated with stress softening at unloading, the permanent set after unloading, the induced anisotropy, as well as the recovery at heating, etc. (detail will be given in §2). Because of complex coupling with very large rubberlike elastic deformations, comprehensive modeling of these features has been and remains a challenging topic. In the past decades, attention has been focused on modeling the widely known feature in isothermal case, namely, the Mullins effect with stress softening. Numerous significant results have been derived in this respect. Here, only representative samples of references are mentioned. Detail may be referred to a most recent review article by Diani et al. (2009).

In majority of the existing studies, a damage variable, γ, is introduced and a certain scalar quantity, u, is also introduced to govern the evolution of this damage variable, in conjunction with a criterion to define two different kinds of responses, namely, the loading response and the unloading response. Then, a pseudo-elastic potential, W, is obtained by multiplying

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[*] State Key Laboratory of Advanced Special Steels, Shanghai University, China
E-mail address: xiaoheng@shu.edu.cn.

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the classical elastic strain energy function $W_0(\boldsymbol{F})$ by a reduction factor, $(1-\gamma)$, namely, $W=(1-\gamma)W_0(F)$ (here $\boldsymbol{F}$ is the deformation gradient) and, finally, the stress response is derived from this pseudo-elastic potential as in the case of classical hyperelasticity. This approach was initiated by Simo (1987) and developed later on. A number of models with different forms of the damage variable γ associated with the quantity u were proposed in, e.g., Lion (1996), Beatty and Krishnaswamy (2000), Laiarinandrasana, et al. (2003) and Chagnon et al. (2004) with the damage variable γ keeping constant in the unloading response, as well as Ogden and Roxburgh (1999), Dorfman and Ogden (2004), Diani et al. (2006a), and Li et al. (2008) with the damage variable γ evolving in the unloading response. According to these models, stress-strain curves at unloading and at subsequent reloading coincide with each other. Toward representing different unloading and reloading curves associated with viscous effects, it was suggested by Miehe (1995) that the damage variable may be partly governed by an arclength quantity resulting from a thermodynamic force driving the damage change. This idea was later developed by, e.g., Miehe and Keck (2000), Lin and Schomburg (2003) and others. Furthermore, Besdo and Ihlemann (2003a) proposed a model with the stress-strain response defined by two asymptotic curves evolving with the deformation history.

Other approaches were suggested by Qi and Boyce (2004) based on the concept of a two-phase system combining a hard and a soft phase and also by De Tommasi et al. (2006) based on the concept of a two-phase material with two natural reference configurations. On the other hand, micro-mechanical models were proposed based on macromolecular mechanisms. Earlier, an isotropic macromolecular three-chain model was established by Govindjee and Simo (1991). Later, this model was developed by Göktepe and Miehe (2005) to account for the induced anisotropy and the residual strain. Chain-like macromolecular models with cross-linking networks were further developed by considering various micro-structural features of macromolecular networks responsible for softening effects, such as molecule slipping, weakening cross-links, chain disentanglement, breaking bonds, damage with void growth, etc. For recent samples of such models, reference may be made to Marckmann et al. (2002), Besdo and Ihlemann (2003b), Diani et al. (2006b), Itskov et al. (2010), Zaïri et al. (2011), Dargazany et al. (2014) and many others. Furthermore, significant results have been presented in modeling either the rate-dependent behavior or the cyclic behavior of elastomers in most recent studies by Drozdov and Dorfman (2003), Colak (2005), Drozdov (2007), Dusunceli and Colak (2008), Ghorbel (2008), Ayoub et al. (2010, 2011a,b), Drozdov et al. (2013), Cantournet et al. (2009), Ayoub et al. (2014), Rodas et al. (2014) and others.

In recent years, numerous studies have been done in the active area of modeling the thermo-mechanical behavior of shape memory polymers. Representative samples of most recent results may be found, e.g., in Liu et al. (2006), Chen and Lagoudas (2008), Kafka (2008), Qi et al. (2008), Kim et al. (2010), Baghani et al. (2012) and many others for thermo-mechanical constitutive models from various standpoints. Further results for healing and damage effects may be found in the most recent contributions by Voyadjis et al. in, e.g., Vojadjis et al.(2011, 2012a, b) and Shojaei and Li (2014).

With numerous results as indicated above, however, it appears that no general agreement has been reached on the modeling of the Mullins effect, and issues and limitations concerning various models may be recognized, as indicated in Diani (2009). In particular, it may be noted that the thermal recovery at heating has not been treated yet. In general, comprehensive modeling of the main non-elastic features of elastomers, including the stress softening, the induced anisotropy, the permanent set as well as the thermal recovery etc., remains a challenge.

In this study we are going to demonstrate that the foregoing non-elastic features of elastomeric materials may be derived as direct, natural consequences of finite elastoplastic flows dominated by large rubberlike elastic deformations. Specifically, the objective of the present study is fourfold, namely, (i) to indicate how rubberlike elastic behavior may evolve with development of plastic flow, (ii) to study how rate-dependent effects may be characterized to ensure smooth transitions to the rate-independent case, (iii) to demonstrate how plastic flow may be induced in a process of pure heating, and, finally, (iv) to propose thermo-coupled, rate-independent and rate-dependent combined hardening J_2-flow models with evolving rubberlike elasticity and then show how typical inelastic features observed in soft solids such as rubberlike materials may be derived as direct, natural consequences of these models.

The above objectives represent novel ideas both in modeling non-elastic behavior of soft solids and in broadening the scope of elastoplasticity. These ideas are motivated just by the typical non-elastic deformation feature observed in elastomers, namely, small irrecoverable deformations (residual strains or permanent sets) are coupled with large recoverable deformations. It is suggested that this feature turns out to be just the feature of finite elastoplastic flows dominated by large elastic deformations. As a consequence, the scope of elastoplasticity may be broadened to cover complex inelastic behavior of elastomers. It may be interesting to note that this new development is in sharp contrast with usually treated metal elastoplasticity in the following sense: very small elastic deformations coupled with large irrecoverable deformations are characteristic of *hard solids* such as monocrystalline and polycrystalline metals, whereas large elastic deformations coupled with very small irrecoverable deformations are typical of *soft solids* such as rubberlike materials. The former is widely known and elastoplasticity models of such nature have been extensively studied to date, as may be seen in, e.g., the classic monograph by Hill (1950) and the modern monograph by Khan and Huang (1995). However, it does not appear that elastoplasticity models with large rubberlike elastic deformations have been sufficiently developed and studied.

In addition to the novelties indicated above, also we are going to show that a comprehensive simulation of the main inelastic features of elastomers may be achieved in an explicit, accurate sense, namely, the multiaxial constitutive functions and parameters incorporated in the proposed model may be determined by means of direct, explicit procedures, so that any given uniaxial tensile and compressive data in a broad sense (see §2) may be automatically, accurately fitted over the entire deformation range, thus bypassing usual complexities involved in identifying unknown constitutive functions and