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A new viscoelastic model based on generalized method of cells for fiber-reinforced composites

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A semi-analytical model is constructed to investigate the relaxation properties of polymer composites consisting of linearly viscoelastic matrices and transversely isotropic elastic fibers. A representative unit cell is subjected to some prescribed loadings to value the time-dependent properties of composite materials. The model is derived to describe the anisotropic viscoelastic properties of composite materials as functions of matrix and fiber properties. The present numerical algorithm is used to predict the viscoelastic properties of a graphite/epoxy composite and the results are compared with the finite element analysis of the micromechanical model. Very good agreement between numerical algorithm and finite element analysis results is illustrated for the viscoelastic properties of anisotropic composite materials.

1. Introduction

Carbon fiber reinforced polymer matrix composites, whose performances continue to improve, have been developed rapidly and used in conventional and novel areas, including aerospace, civil, electronic and medical engineering (Khan and Yeakle, 2011; Jia et al., 2012; Li and Yu, 2006). These materials are known to exhibit viscoelastic behavior, especially at high temperature and moisture, in which the magnitude of the stress components is a function of the deformation history and it depends on strain, strain rate, temperature and time. So, one of the most fundamental problems for the designer in the mechanics of composite materials is to predict effective properties based upon constituent (fiber and matrix) properties, spatial arrangement (packing), and constituent volume fractions.

The micromechanical methods provide efficient tools to evaluate the behavior of the composite materials. Many researchers have devoted considerable effort to characterize macro-mechanical properties of composites by using micromechanics modeling method and many micromechanics models have been proposed in the literature during the past fifty years and this trend continues (Malcher et al., 2014; Zhang and Waas, 2014; Hachour et al., 2014; Aghababaei and Joshi, 2011, 2013; Ayoub et al., 2013; Lahellec and Suquet, 2013; Chen and Ghosh, 2012; Dai and Ng, 2012; Brassart et al., 2012; Azizi et al., 2011; Kruch and Chaboche, 2011; Lee et al., 2011; Horstemeyer and Bammann, 2010; Le Quang and He, 2007; Williams, 2005). The classical concentric cylinder models based on the ellipsoidal/cylindrical inclusion analysis have been summarized in the works of Christensen (1979) and Nemat-Nasser and Horii (1993), and extended to the inelastic domain for idealized geometries and limited loading conditions by Pindera et al. (1993), Williams and Pindera (1997). This has motivated the development of models based on various assumptions regarding the local state of stress or strain in the individual

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phases of a heterogeneous material. Mori–Tanaka scheme (Mori and Tanaka, 1973), in which the state of stress and strain in the entire matrix phase is taken to be constant, continues to be used in different forms to this day (Peng et al., 2013; Jain et al., 2013; Lu, 2013; Mortazavi et al., 2013). Chaboche et al. (2001) have generalized this approach and demonstrated to capture with good accuracy the local fields relative to finite-element simulations. Another micromechanics approach which also uses the average field theory, proposed formerly by Eshelby (1957), by taking into account the average micro-stress and micro-strain fields to estimate the macroscale mechanical response of the heterogeneous medium is self-consistent methods (Huang et al., 1994; Hill, 1965). These homogenization methods have been extensively used in the linear analysis for composite structures and also found utility in case of damage and failure analysis by extending their utility to the nonlinear regime (Prabhakar and Waas, 2013). However, as based on the homogenization technique and the average of the constituent properties, the main shortcoming of these analytical methods is their failure in simulation of the complex structures, too stiff result for inelastic responses (Chaboche et al., 2005) and their inability to capture precise stress redistribution in an inelastic analysis (Masson and Zaoui, 1999). To resolve these issues, several models, such as "tangent" (Doghri et al., 2010, 2011), "secant" (Berveiller and Zaoui, 1978) and "affine" (Masson et al., 2000; Masson and Zaoui, 1999) models, are developed in the literatures. The microstructure of a composite material is represented by a repeating unit cell (RUC), which can be subsequently partitioned into a number of sub-regions. The method of cells (MOC) (Aboudi, 1991), and its extension, and the generalized method of cells (GMC) (Paley and Aboudi, 1992; Pineda et al., 2013) employing a combination of piecewise uniform stress and strain fields as discussed in more detail in the sequel are other powerful semi-analytical methods to approximate the composite effective behavior. In the GMC, the RUC is divided into an arbitrary number of subcells to account for the effect of fiber geometry and packing arrangement. The continuity conditions of the displacement and traction at the interfaces between subcells and between adjacent RUCs are imposed on an average basis, resulting in a set of equations that relate the local microscopic strains to the global macroscopic strains by a concentration tensor, thus the local strain fields can be solved by knowing the applied fields. The local stress fields are readily resolved from the local constitutive relations, and the composite effective properties are determined from the local fields via a volume average. The significant improvement of GMC is high-fidelity generalized method of cells (HFGMC) (Aboudi et al., 2001, 2003; Haj-Ali and Aboudi, 2009; Bednarcyk et al., 2004) which employs much better representations of stress and strain fields in the individual phases unlike the iso-strain and iso-stress approach of GMC. HFGMC employs arbitrary constitutive theories for modeling local phase response and produces closed-form macroscopic constitutive equations for the homogenized material response, just the same as GMC. However, it is not easily implementable, efficient or capable of modeling realistic microstructures. The computational time increases rapidly if more detail of the nonlinear effects in the local fields is required to be captured.

In the analysis and characterization for viscoelastic response of unidirectional composite, using finite element method analyses (FEA) to solve time-dependent problem, especially with long-term loading, is difficult and requires numerous computational resource since the constitutive equation of viscoelastic materials is in hereditary integral form. The accuracy of FEA result strongly depends on time steps. To avoid such problems, a convolution theorem of Laplace transform is employed. Hashin (1972, 1983) firstly discussed the micromechanical modeling using the correspondence principle. By using correspondence principle, all of computational procedures are done in Laplace domain. Then, the solution can be obtained by applying the inverse transformation (Yancey and Pindera 1990; Matzenmiller and Gerlach 2004; Shrotriya and Sottos, 2005; Huang et al., 2011; Barai and Weng, 2009; Aldraihem, 2011). Several other computational algorithms and constitutive models have been forwarded for analysis and prediction of the overall behavior of composite materials (Matzenmiller and Gerlach, 2004; Chan et al., 2006; Naik et al., 2008; Nedjar, 2011). However, the Laplace transformation technique can save an expansive computation amount and reduce the mortal formulation of numerical procedure. The accuracy of time-dependent solution depends on the precision of the mechanical model and algorithm which are used in Laplace domain.

In this work, a semi-analytical model is established to determine the properties of unidirectional fiber-reinforced composites consisting of linearly viscoelastic matrix and transversely isotropic elastic fibers, based on the generalized method of cells. The micromechanical approach is used to deduce the analytical expressions in time domain. By introducing a factor which can be adjusted according to the specific situation (i.e. the fiber distribution, loading condition and geometries of fibers), the model provides an effective procedure to investigate the macroscale mechanical behaviors of viscoelastic composite materials. A RUC model is subjected to some axial and shear loadings to study and quantify the time-dependent properties of composite materials. The accuracy of the model and influence of the parameter factor on the results are assessed by comparison against corresponding finite element analyses.

2. Semi-analytical model

The GMC, proposed by Paley and Aboudi (1992), is employed to predict the concentration tensors, which has been proven to be very efficient in representing the elastic and inelastic behaviors of fiber-reinforced unidirectional composite materials (Aboudi, 1996). In the two dimensional formulation, the composite consists of continuous fibers in the x_1-direction, which are arranged in a square fiber array packing in the x_2–x_3 plane (Fig. 1(a)). Due to this assumption it is possible to identify a unit cell as a RUC with circular cross section of fiber embedded. Furthermore, in the GMC models the cross-section of the fibers is assumed to be rectangular (Paley and Aboudi, 1992; Aghdam and Dezhsetan, 2005). In this work, composite properties are assumed to be instantaneously linear elastic at a given moment and RUC is simplified as the one consisting of four subcells one of which stands for the fiber material, while the other three are occupied by the viscoelastic matrix material as