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Investigation of localized necking in substrate-supported metal layers: Comparison of bifurcation and imperfection analyses

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ABSTRACT

Localized necking is often considered as precursor to failure in metal components. In modern technologies, functional components (e.g., in flexible electronic devices) may be affected by this necking phenomenon, and to avoid the occurrence of strain localization, elastomer substrates are bonded to the metal layers. This paper proposes an investigation of the development of localized necking in both freestanding metal layers and elastomer/metal bilayers. Finite strain versions of both rigid–plastic flow theory and deformation theory of plasticity are employed to model the mechanical response of the metal layer. For the elastomer, a neo-Hookean constitutive law is considered. Localized necking is predicted using both bifurcation (whenever possible) and Marciniak–Kuczynski analyses. A variety of numerical results are presented, which pertain to the prediction of localized necking in freestanding metal layers and metal/substrate bilayers. The effects of the constitutive framework and the presence of an elastomer substrate on strain localization predictions have been specifically highlighted. It is demonstrated that the addition of an elastomer layer can retard significantly the occurrence of localized necking. It is also demonstrated that the results of the Marciniak–Kuczynski analysis tend towards the bifurcation predictions in the limit of a vanishing size for the geometric imperfection.

1. Introduction

Nowadays, necking limit prediction of metal sheets or thin plates represents an ambitious challenge both for the sheet metal forming industry and for the design of functional components in electronic devices. In the well-known experiment of a tensile test on a metal specimen, the deformation develops mainly through three successive stages: (i) a homogeneous deformation; (ii) a progressively concentrating strain under a constant or smoothly decreasing load (diffuse necking), and (iii) an abrupt strain localization (localized necking) under a rapid load decrease. The onset of localized necking represents the ultimate deformation that a stretched metal sheet can undergo, since this phenomenon is often precursor to material failure. Probably, the most common representation of this limit is through the concept of forming limit diagram (FLD). Note that this concept was initially introduced in the beginning of the sixties by Keeler and Backofen (1963), in the range of positive minor principal strains (i.e. $\varepsilon_2 > 0$), and Goodwin (1968) (extending Keeler's work to negative minor principal strains, i.e. $\varepsilon_2 < 0$). In the literature, a large amount of studies have been devoted to the experimental and numerical determination of FLDs for sheet metals with different material properties (Smith and Lee, 1998; Narayanasamy and Sathiya, 2005; Strano and Colosimo, 2006; Khan and Baig, 2011; Zhang and Wang, 2012; Li et al., 2013). In the vast majority of these studies, attention

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was restricted to freestanding metal sheets. However, the need for increasing the ductility of metal components has induced an impetus to develop alternative and more complex materials such as substrate-metal bilayers. Indeed, supporting a metal layer by an elastomer substrate has proven to significantly improve its ductility (Chiu et al., 1994; Hommel and Kraft, 2001; Alaca et al., 2002) and to enhance its energy absorption (Xue and Hutchinson, 2007; Xue and Hutchinson, 2008). In an industrial context, substrate-supported metal layers are being used in a variety of flexible electronic devices such as conductors and interconnects (Lacour et al., 2005; Cotton et al., 2009; Cordill et al., 2010; Graudejus et al., 2012). Despite the increasing industrial interest in elastomer/metal bilayers, there is still a need for further studies for an in-depth understanding of the corresponding strain localization conditions. In this paper, an attempt is made to investigate the impact of an elastomer layer, bonded to a metal sheet or plate, on the shape and location of the associated FLD in the in-plane principal strain space. Note that the concept of FLD and associated terminology, more commonly adopted in the literature for a freestanding metal layer, is extended here to a metal/substrate bilayer. In the related literature (Xue and Hutchinson, 2007; Jia and Li, 2013), a similar terminology, namely "necking limit diagram", is preferred instead.

Due to the complexity of the experimental determination of the FLD and its relatively high cost, a number of theoretical and/or numerical models have been set up. These alternative approaches require the use of a criterion, to predict the occurrence of strain localization, along with a constitutive law to describe the evolution of the mechanical state of the studied material. The onset of plastic flow localization may occur as a bifurcation from a homogeneous deformation state or it may be triggered by some assumed initial imperfection. Accordingly, two main classes of strain localization criteria, which will also be used in this paper, can be found in the literature:

- Imperfection approach:
  This approach postulates the existence of an initial imperfection in the form of a narrow band across the studied metal layer. This imperfection may be assumed as local variations in thickness (geometric imperfection) or in plastic properties (material imperfection), which will affect the plastic flow and therefore influence the strain localization occurrence. This approach was initially introduced by Marciniak and Kuczynski (1967), which will be referred to hereafter as the M–K analysis. In this pioneering work, the authors introduced an initial geometric imperfection in the sheet plane in the form of a groove or band. During in-plane biaxial stretching, the plastic deformation concentrates in the band more than in the rest of the sheet, leading thus to localized thinning in the band. In its initial version, the M–K analysis was restricted to a groove perpendicular to the major strain direction, thus limiting the prediction of FLD to the right-hand side of the FLD ($\varepsilon_2 > 0$). To overcome this limitation, Hutchinson and Neale (1978b) proposed an extension of this approach. This extension covers the full range of the FLD by considering all possible initial orientations of the groove and selecting the lowest value of the major strain at the onset of localized necking as the limit strain. The accuracy in the prediction of the FLD has increased over the years by improvement of the constitutive modeling (see, e.g., Barlat, 1989; Eyckens et al., 2009). In spite of the over-sensitivity of its predictions to the initial imperfection value (see, e.g., Baudelet, 1984), the M–K analysis has attracted a great deal of attention, due to its pragmatic character.
- Bifurcation analysis:
  In many ductile materials, zones of localized deformation are commonly observed prior to failure, which are considered as a result of instability in the constitutive description of homogeneous deformation. These localization bands induce a macroscopic discontinuity in the velocity gradient of the deforming material and often signal the inception of failure. In addition to its sound mathematical background, the bifurcation theory does not require any fitting parameter, such as the initial imperfection needed in the M–K analysis. The bifurcation analysis was initially proposed by Hill (1952) in the case of flow theory of plasticity (rigid–plastic material) within the framework of generalized plane stress. His theory predicts that localized necking occurs along a line of zero extension and is therefore restricted to negative minor strain values (i.e., the left-hand side of the FLD). For elasto-plastic material models with smooth yield surface and associative plasticity, it has been shown (see, e.g., Rice, 1976) that the bifurcation approach does not predict localized necking at a realistic stress level under positive in-plane biaxial stretching (i.e., in the right-hand side of the FLD). In order to overcome this limitation, the introduction of some destabilizing effects is required to promote material instability. To this end, a number of authors suggested that the subsequent yield surfaces of the material would develop a vertex-like structure during continued deformation. The development of such a destabilizing vertex may be due to the application of the deformation theory of plasticity (Stören and Rice, 1975; Hutchinson and Neale, 1978b; Hutchinson and Neale, 1981), or to the use of the Schmid law within the framework of crystal plasticity (Franz et al., 2009; Franz et al., 2013). Material instability may also be due to some softening behavior introduced in the constitutive modeling through coupling with damage (see, e.g., Rudnicki and Rice, 1975; Saje et al., 1982, for pressure-sensitive void containing materials, or Haddag et al. (2009), within the framework of continuum damage mechanics).

The main objective of this paper is to extend the earlier contributions of Hutchinson and Neale (1978b), Xue and Hutchinson (2007) and Jia and Li (2013). In the first of this series of works, Hutchinson and Neale (1978b) extensively studied the necking limit of freestanding metal sheets (or layers) using the bifurcation and the imperfection approaches. For each of these two localization approaches, a rigid–plastic finite strain version of the J_2 flow theory (designated shortly in what follows as "flow theory") and of the J_2 deformation theory of plasticity (called shortly hereafter "deformation theory") were used to model the mechanical behavior of the metal sheet. Our current study extends Hutchinson and Neale (1978b) work to the case of substrate-metal bilayers. The constitutive models of the metal layer are taken the same as those used in Hutchinson and Neale