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# Rotary inertia and temperature effects on non-linear vibration, steady-state response and stability of an axially moving beam with time-dependent velocity

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**Abstract**

Free non-linear transverse vibration of an axially moving beam in which rotary inertia and temperature variation effects have been considered, is investigated. The beam is moving with a harmonic velocity about a constant mean velocity. The governing partial-differential equations are derived from the Hamilton's principle and geometrical relations. Under special assumptions, the two partial-differential equations can be mixed to form one integro-partial-differential equation. The multiple scales method is applied to obtain steady-state response. Elimination of secular terms will give us the amplitude of vibration. Additionally, the stability and bifurcation of trivial and non-trivial steady-state responses are analyzed using Routh-Hurwitz criterion. Eventually, numerical examples are presented to show rotary inertia, non-linear term, temperature gradient and mean velocity variation effects on natural frequencies, critical speeds, bifurcation points and stability of trivial and non-trivial solutions.


*Keywords:* Non-linear vibration; Stability; Axially moving beam; Bifurcation; Multiple scales method

## 1. Introduction

Axially moving beam models may be used for many engineering devices, e.g., paper sheets, belts, fiber textiles, band saw blades, magnetic tapes and furnace conveyor belts all are classified as axially moving beams.

Moving beams can be modeled as linear or non-linear. Velocity can be constant or harmonically variable. Time-dependent transport velocity means a mean velocity plus small periodic fluctuations.

In fact, many real mechanisms can be represented as axially moving beams with time-dependent velocity.

There are many researches which have been carried out on axially moving systems in literatures. Mote [1] did not consider flexural stiffness and carried out earliest investigation about critical speed on non-linearity of system. Ames [2] examined the motion of moving thread line under planar periodic boundary excitation. He showed that the plane motion is non-linear and hard spring jump is due to critical velocity effect. Thurman and Mote [3] used perturbation method to measure the relationship between linear and non-linear fundamental period deviation and speed. Tabarrok et al. [4] obtained four non-linear partial-differential equations from modeling a beam whose length changes with time. Wickert and Mote [5] used symmetric and skew symmetric differential operators to illustrate equations of motion. Then, they used the Green function method and obtained critical transport speed. Wickert [6] analyzed free non-linear vibration of an axially moving beam in sub and super critical transport velocity range. Stylianou and Tabarrok [7] used finite element analysis to obtain numerical solutions. Then, they examined effects of tip mass and high frequency of axial motion fluctuations to transverse vibration. Pakdemirli et al. [8] investigated the transverse vibration of an axially moving string. He used the

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**Nomenclature**

| | |
|---|---|
| $r(x,t)$ | longitudinal displacement of the beam |
| $w(x,t)$ | transverse displacement of the beam |
| $\psi(x,t)$ | angle of rotation due to bending |
| $\rho A$ | constant mass per unit length |
| $\alpha$ | heat expansion coefficient |
| $\mathrm{d}T$ | temperature variation |
| $U$ | potential energy |
| $T$ | kinetic energy |
| $\varepsilon_L$ | non-linear strain |
| $\omega$ | frequency of varying speed |
| $v_0$ | mean velocity |
| $\varepsilon v_1$ | amplitude of fluctuations |
| $\sigma$ | detuning parameter |
| $\omega_n$ | $n$th natural frequency |

Galerkin method and discretized the equations. The results show that instabilities occur at much higher amplitudes. Oz et al. [9] investigated the transition behavior between string and beam. They used perturbation analysis and constructed an outer solution for a variable speed beam. Stylianou and Tabarrok [10] used finite element method to examine effects of wall flexibility, damping and tip support on stability of system. Oz and Pakdemirli [11] obtained the response of an axially accelerating, elastic, pretensioned beam. He used the multiple scales methods to solve equations of motion and finally investigated principal parametric resonances in detail. Chakraborty et al. [12] studied free and forced responses of a traveling slender beam including the non-linear terms. He calculated the response of the beam, excited by a point harmonic load. Pakdemirli and Ozkaya [13] used the method of multiple scales and obtained a boundary layer solution for constant speed beam. Parker [14] examined the stability of an axially moving string supported by a discrete elastic foundation. He found that this system is in class of dispersive gyroscopic continua. Pellicano and Zirilli [15] analyzed oscillation of an axially moving beam with vanishing flexural stiffness and weak non-linearities. Oz et al. [16] investigated non-linear vibration of an axially moving beam under harmonically varying velocity. They used the multiple scale method to obtain solvability condition and amplitude of vibration for the Euler–Bernoulli beam theory. Chen et al. [17] obtained bifurcation diagram versus dynamic viscosity, transport speed and periodic perturbation for string. Shin et al. [18] examined the out-of-plane vibration of membrane using the Hamilton's principle and Galerkin method to discretize equations and investigate effects of system parameters on natural frequencies, mode shapes and stability of system. Chen et al. [19] used modified finite difference method to solve the transverse vibration equations. Zhang and Chen [20] investigated non-linear behavior of axially moving viscoelastic string. Chen and Zhao [21] investigated transverse vibration of axially moving beam with a low axial speed. Chen and Yang [22] considered an axially moving viscoelastic, Euler–Bernoulli beam with time-variant velocity. They used only strain which is caused by bending moment and neglected strain which is made by gradient of longitudinal displacement. Kartik and Wickert [23] investigated forced vibration of axially moving strip which is guided by a partial elastic foundation and edge imperfection. Sze et al. [24] used incremental harmonic balance method to solve discretized equations of motion of an axially moving beam under constant speed.

Recently, variable speed non-linear beams with Euler–Bernoulli beam theory has been considered.

In present investigation, a harmonically varying speed non-linear beam with rotary inertia and temperature variation effects is considered. The governing equations are coupled from which an integro-partial-differential equation obtained. Applying multiple scales method, stability and bifurcation of non-trivial and trivial steady-state response for frequency of variable transporting speed close to twice of system natural frequency are analyzed using Routh-Hurwitz criterion. Numerical examples helped us to show the effect of rotary inertia, temperature variation, non-linear term and mean velocity on natural frequencies, critical speeds, bifurcation points and stability of trivial and non-trivial solutions. Finally frequency–response curves are drawn.

## 2. Equations of motion

A beam with axial stiffness of $EA$ and the flexural rigidity of $EI$ is shown in Fig. 1. Additionally, this beam is under applied pretension, $N$, temperature variation, $\mathrm{d}T$, and a harmonically varying transport speed, $v$, between simple supports at each end. As shown in Fig. 1, $r(x,t)$ and $w(x,t)$ describe longitudinal and transverse displacements of the beam.

There are some simplifications as below:

- shear deformation is neglected, namely only Euler–Bernoulli theory with rotary inertia effect is considered;
- displacements are considered in two dimensions not three. It means, the out-of-plane motion is not considered;
- variations of cross-sectional dimensions are assumed to be negligible;
- the axial stiffness is supposed to be large enough to neglect deformation resulting from pretension;
- the longitudinal displacement is small enough to neglect non-linear strain deriving from $r(x,t)$;
- temperature is varying linearly and the strain caused by temperature variation is in longitudinal direction and in other directions, it is supposed to be released.