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Effect of rotation and thermal relaxation on Rayleigh waves in piezothermoelastic half space

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Abstract

The present paper represents an analysis of Rayleigh surface waves in a homogeneous, transversely isotropic, generalized piezothermoelastic half-space rotating with uniform angular velocity about normal to its boundary and subjected to stress free, electrically shorted/charge free and thermally insulated/isothermal boundary conditions. The secular equations for stress free, generalized piezothermoelastic half-space in closed form and isolated mathematical conditions are derived. The characteristics of surface waves propagating in generalized piezothermoelastic solid half-space and their dependence upon various geometric and physical parameters have been investigated. Finally, in order to illustrate and compare the theoretical results, numerical solution of various secular equations and other relevant relations are derived for cadmium selenide (6 mm) class material by adopting functional iteration scheme after employing Descartes' algorithm to compute the characteristic roots of coupled differential equation system. The corresponding simulated results of various physical quantities such as phase velocity, attenuation, specific loss, frequency shifts and electromechanical coupling have been presented graphically for rotation and non-rotation cases. The study will be useful in design and construction of gyroscope, rotation sensors, temperature sensors and other pyro/piezoelectric SAW devices.

Keywords: Piezothermoelasticity; Electrically shorted; Rotation; Surface waves; Thermal relaxation

1. Introduction

In recent years, piezoelectric materials have been integrated with structural systems to form a class of "smart structures". The piezoelectric materials are capable of altering the structure's response through sensing, actuation and control. Piezoelectricity can be used in many other different applications. Piezoelectricity literally means pressure electricity. This phenomenon of piezoelericity was first discovered by Pierre and Jacques Curie [1]. The development of quartz transducers to generate and detect underwater acoustic waves has been successfully used for submarine detection. One of the most important applications of piezoelectricity was radio communications, which was finally put into use in the form of first quartz crystal controlled transmitter. The thermo-piezoelectricity theory was first proposed by Mindlin [2] and the governing equations of thermo piezoelectric plate were also derived by Mindlin [3]. The effect of rotation on wave characteristics in piezoelectric crystals has been discussed by various authors such as Gates [4] and Soderkvist [5]. According to Wren and Burdess [6] and Clarke and Burdess [7], the surface acoustic waves (SAWs) in elastic solids are greatly affected by rotation and the speeds of disturbed waves are dependent upon rotation rate. This can be made applicable to design rotation rate sensors based on SAW. Fang et al. [8] studied rotation-perturbed SAWs propagating in piezoelectric crystals. Fang et al. [9] also studied the rotation sensitivity of waves propagating in a rotating piezoelectric plate. These rotation sensitivity characteristics are helpful in the development of rotation sensors and other piezoelectric devices for which frequency insensitivity to rotation is desired. Yang et al. [10] discussed the thickness vibrations of rotating piezoelectric plates. Teston et al.

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[11] studied acoustic electric effect due to conductive liquid loading on the Lamb waves in piezoelectric composite. It is noticed that the fundamental acoustic mode shows a low phase velocity and a high electromechanical coupling factor. It is also shown that in case of piezocomposites, the permittivity model takes into account the strong interaction between the acoustic wave and the liquid. Sharma and Pal [12] and Sharma et al. [13] investigated the propagation of Lamb waves in piezothermoelastic materials in the context of coupled thermoelasticity. Sharma and Walia [14] studied the straight and circular crested waves in generalized piezothermoelastic plates. Sharma and Thakur [15] investigated the effect of rotation on Rayleigh–Lamb waves in magneto-thermoelastic media. The propagation of thermoelastic surface waves has been investigated in detail by Nayfeh and Nasser [16] in the context of linear generalized thermoelasticity. Earlier, Rayleigh [17] studied propagation of waves along with the plane surface on an elastic solid. Bluestein [18] studied a new surface wave in piezoelectric materials. Clarke and Burdess [19] investigated Rayleigh waves on a rotating surface. Sharma et al. [20] investigated the problem of propagation characteristics of Rayleigh waves in transversely isotropic piezothermoelastic materials. In order to remove the paradox of infinite velocity of thermal disturbance, different authors [21–24] modified the Fourier law of heat conduction and constitutive relations so as to obtain hyperbolic equation for heat conduction that led to the development of non-classical theories of thermoelasticity. These models include the time lag needed for the acceleration of heat flow and take into account the coupling between temperature and strain fields. The existence of "second sound" is also supported by experimental [25–27] exhibition. Chadwick and Windle [28], Chadwick and Atkin [29], Lockett [30] and Nayfeh and Nasser [31] investigated the propagation of Rayleigh waves along isothermal and insulated boundaries. Chandrasekharaiah [32,33] developed the generalized theory of thermo-piezoelectricity by taking into account the finite speed of propagation of thermal disturbance. Sharma and Walia [34] studied Rayleigh waves in piezothermoelastic materials in the context of generalized theories of thermoelasticity. The detailed studies and analysis of piezoelectric vibratory gyroscopes can be found in the recent publications by Yang and coworkers [35–38]. Yang [39] brought out a review of some topic on piezoelectricity. The Stroh formalism, a complete variable technique for two-dimensional elasticity was used in Ting [40].

It is well established that Rayleigh waves travel along the surface of relatively thicker solid materials penetrating to a depth of one wavelength and therefore, the study of these waves is very useful because of their sensitivity to surface defects. Moreover, since they follow the surface around, so these waves can also be used to inspect areas that other waves might have difficulty in reaching. Keeping all this in view, an attempt has been made to study the effect of thermal relaxation and rotation on Rayleigh wave characteristics in piezothermoelastic transversely isotropic half-space. The secular equations governing Rayleigh surface waves in closed form and isolated mathematical conditions are derived. According to Ting and Minzhong [41], although the Stroh formalism is most elegant when the boundary conditions are simple yet in case of mixed boundary conditions the concise matrix expressions of the Stroh formalism are destroyed. We have used a hybrid algorithm consisting of direct method (Descartes' Method) for solving fourth degree complex polynomial equation to compute the characteristics roots involved in the formal solution which are then utilized in the secular equation to compute wave characteristics such as phase velocity and attenuations coefficients with the help of functional iterative numerical technique. The numerically simulated results have been presented graphically for CdSe material solid half-space.

2. Basic equations

The basic governing dynamical equations of linear generalized piezoelectric thermoelastic interactions in homogeneous anisotropic solid with Coriolis and Centrifugal forces are [14,20,32,33]

(A) Strain–displacement relations

$$S_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \quad i,j = 1,2,3. \tag{1.1}$$

(B) Stress–strain–temperature and electric field relations

$$\sigma_{ij} = c_{ijkl}S_{kl} - \beta_{ij}(\dot{T} + t_0\delta_{1k}\ddot{T}) - e_{kij}E_k \quad (i,j,k,l = 1,2,3). \tag{1.2}$$

(C) Equation of motion

$$\sigma_{ij} + \rho F_i = \rho\left[\ddot{u}_i + 2\varepsilon_{ijk}\Omega_j\dot{u}_k + \left(\Omega_i\Omega_j u_j - \Omega_j\Omega_j u_i\right)\right], \quad i,j = 1,2,3. \tag{1.3}$$

(D) Heat conduction equation

$$K_{ij}T_{,ij} = T_0(\beta_{ij}\dot{u}_{i,j} - p_i\dot{\phi}_{,i}) + \rho C_e(T + t_1\delta_{2k}\dot{T}), \quad (i,j = 1,2,3). \tag{1.4}$$

(E) Gauss equation

$$D_{i,j} = 0. \tag{1.5}$$

The electric displacement $D_i = e_{ijk}S_{jk} + \in_{ij}E_j + p_i(\dot{T} + t_0\delta_{1k}\ddot{T})$, $(i,j,k = 1,2,3)$, the electric field $E_i = -\phi_{,i}$ and $\vec{\Omega} = (\Omega_i)$ is the uniform angular velocity of rotation and t_1, t_0 are thermal relaxation times and δ_{ik} is the Kronecker's delta in which $k = 1$ corresponds to Lord–Shulman (LS) theory and $k=2$ refer to Green–Lindsay (GL) theory of thermoelasticity: According to Green [42], the thermal