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Confidence bounds on component reliability in the presence of mixed uncertain variables

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Abstract

Uncertainties in a physical system can be modeled and analyzed by using probability theory or possibility theory, depending on the amount of information available. In probability theory, uncertain variables are modeled using probability density functions (PDFs) and then propagated through the system to obtain its reliability. In the absence of sufficient data to model a PDF, possibility theory, in which variables are represented using fuzzy membership functions, can be used to propagate uncertainty. However, when dealing with a combination of both probability distributions and fuzzy membership functions, the computational cost involved in estimating the membership function of reliability increases exponentially because one reliability analysis, which is a computationally expensive procedure, is performed at each possibility level to obtain the bounds on the reliability of the structure. To improve the computational efficiency, a technique that uses response surface models and transformations of possibility functions is presented in this paper. The efficiency and accuracy of the proposed methodology is demonstrated using numerical examples.
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Keywords: Reliability; Membership function; Mixed variables; Fast Fourier transforms; Convolution integral

1. Introduction

Uncertainties present in the design process need to be quantified and propagated to obtain the reliability of a structural system. They can be classified as aleatory uncertainty and epistemic uncertainty. Aleatory uncertainty is the inherent variation in the system whereas epistemic uncertainty is the variation due to lack of knowledge in the system. Typically, there is ample information to model aleatory uncertainty as random variables and epistemic uncertainty can be modeled using non-random variables like possibility membership functions or intervals. When there is enough data about a particular quantity, a probability distribution can be assigned, and this random uncertainty can be propagated using existing probabilistic methods. In situations where sufficient information is not available for defining a probability distribution, fuzzy theory can be used to represent the available data in an analytical form. Using fuzzy theory, these variables can be represented by membership functions based on their possibility of occurrence or level of confidence. Once the input variables are defined as possibility functions, the possibility of the response can be estimated. But in most problems, information might be available to represent some variables with a probability distribution and some with a membership function. Therefore, this paper focuses on dealing with problems for which some uncertainties can be quantified using fuzzy membership functions while some are random in nature.

This methodology is developed to provide a capability for analyzing safety of competing designs during the preliminary design stage. During this stage, information about the design variables that is required to model random variation is typically unavailable and requires modeling them as intervals or fuzzy variables. As the design progresses additional information can be used to modify the variable definitions and eventually approach probabilistic system reliability analysis of the final design.

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In probability theory, the failure probability of a structure is obtained by solving the multi-dimensional integral

$$p_f = \int_{\Omega} f_x(X)\,\mathrm{d}X, \tag{1}$$

where p_f is the probability of structural failure and $f_x(X)$ denotes the joint probability density function (PDF) of the vector for the basic random variables, $X = (x_1, x_2, \ldots, x_n)^{\mathrm{T}}$, which represent uncertain quantities such as loads, geometry, material properties, and boundary conditions. Furthermore, Ω is the failure region modeled by the limit-state function or performance function $g(X)$. The failure region is defined by $g(X) \leqslant 0$. Monte Carlo simulation can be used to deal with this multi-fold integration. However, it requires a large number of samples to accurately estimate the small order of structural failure probabilities. To reduce the computational cost, several algorithms [1–4] were developed that make use of surrogate representations of the failure surface and compute the failure probability.

Reliability analysis methods begin with the prediction of the most probable failure point (MPP). The MPP is the point in the design space that has the maximum probability of failure. Once a good estimate of the MPP is obtained, surrogate models representing the failure surface around the MPP can be used to evaluate the failure probability. The accuracy of the estimated probability of failure depends on the validity of the approximation around the MPP of the limit-state function.

The above multi-dimensional PDF integral is also represented using convolution integral as described in basic probability and statistics literature. For a failure surface that is a linear combination of random variables this convolution integral can be evaluated using Fast Fourier transforms (FFT). In order to use FFT, the limit-state function must be available as a separable closed-form expression. Sakamoto et al. [5] used a response surface approximation to get a closed-form expression for a particular implicit limit-state function. Penmetsa and Grandhi [6] used a Two-point Adaptive Nonlinear Approximation (TANA2) at the MPP for obtaining a closed-form expression for a limit-state function. Using FFT, the joint density function of the random variables is obtained efficiently and the failure probability is calculated by integrating this function over the failure region. These methods were developed to handle only random variables.

In the presence of non-random variables, Briabant et al. [7] presented possibilistic approaches for structural optimization and design. They proposed that it is possible to evaluate fuzzy variables using α-cuts or membership levels, as shown in Fig. 1. These α-cuts are different levels of confidence bounds on the variable of interest. 100% confidence represents a deterministic quantity and a 0% confidence represents the widest bounds. Any level between would be represented using the concept of α-cuts. At each level, the variation of an uncertain parameter is defined by a lower and an upper bound. Once the variables are defined

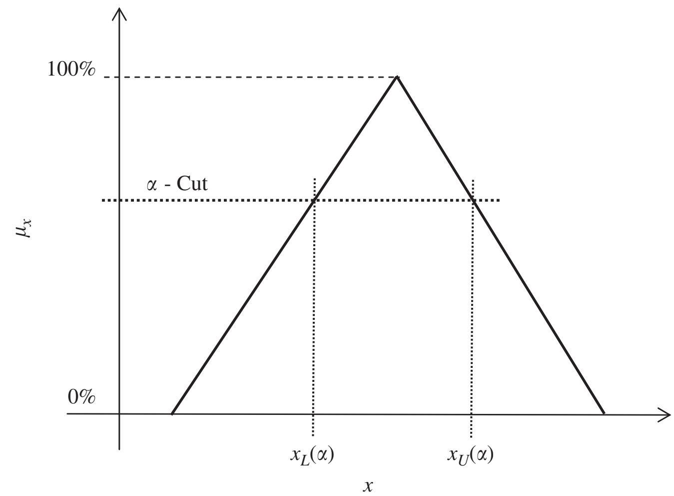

Fig. 1. Membership function showing an α-cut.

as membership functions, the bounds on the response at various α-cuts can be obtained. The Vertex method [8] evaluates the function value at each of the vertices of the design space, represented by the bounds on the variables, to obtain the minimum and maximum values of the response. This method works well for linear problems, but fails to capture the minimum and maximum values for nonlinear non-monotonic responses. For nonlinear problems, the bounds might be present within the design space while the vertex method checks for the function value only at the extremes of the design space. Some of the other methods [9,10] use optimization techniques to calculate the minimum and maximum value of the response within the specified bounds.

All the methods discussed above consider either random variables or fuzzy input, but do not accommodate a combination of variables. Therefore, methods need to be developed for dealing with problems comprising of mixed uncertain variables. Moller et al. [11] introduced a methodology for estimating the membership function of the safety index by considering fuzzy randomness. They formulated a Fuzzy First Order Reliability Method (FFORM) that simultaneously permits the usage of fuzzy variables and random variables. Using this method, the membership function of the reliability index can be estimated. But the calculation of the failure probability from the safety index values is prone to errors.

In the presence of both random as well as fuzzy variables, the computational cost involved in the development of the membership function of reliability increases because multiple reliability analyses, which by itself is a computationally expensive procedure, is required at each confidence level. This is because the entire bounds of the fuzzy variables are to be explored to determine the bounds of the reliability at a particular confidence level. One can reduce the number of reliability analyses at each confidence level by having the information about the configuration of the fuzzy variables that correspond to the extreme values of

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