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Robust stability bounds for nonclassically damped systems with multi-directional perturbations

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Abstract

This paper investigates the stability robustness of nonclassically damped systems with multidirectional perturbations. Bounds on uncertain parameters that maintain the stability of an asymptotically stable, linear multi-degree-of-freedom system with nonclassical damping are derived using specific Lyapunov functions. The explicit nature of the construction permits us to directly express the algebraic criteria in terms of physical parameters of the system. Numerical examples are given to illustrate the effect of the proposed approach.
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Keywords: Robust stability; Lyapunov approach; Algebraic criteria; Classical damping; Multidirectional perturbations

1. Introduction

Nowadays, product departments in automotive industry are using Finite Element models intensively to analyze and solve a variety of engineering problems. These large numerical models are deterministic, in that it is implicitly assumed that all parameters are precisely known and that the manufacturing process produces identical structures. These assumptions are often not met, for example, uncertainty exists on the level of model inaccuracy and physical properties in an early design stage, when design decisions must still be taken, so that dimensions and material properties are not yet fixed. Such an uncertainty should not be described in a probabilistic way because there is not enough information available, so that assigning a probability density function is a subjective change of the problem definition. Uncertainties can lead to severe degradation in performance and even dynamical instability which is of great practical significance and usually an undesirable feature of the behavior of practical systems. It is therefore essential to choose the parameters so as to avoid the possible occurrence of unstable behavior. Hence, the problem of maintaining the stability of a nominal stable system subjected to parametric perturbations has been an active area of research for some time.

Most of previous results on robust stability are restricted to bounds on the parameter uncertainties in the state-space models, see, for example, Zhou and Khargonekar [1], Siljak [2], Bien and Kim [3], Gao and Antsaklis [4], Pun et al. [5] and the literature cited therein. Even though any second-order system can be represented as an equivalent first-order system, retaining the model in matrix second-order form has many advantages. For example, by keeping the system in matrix second-order form, symmetry of the mass, damping and stiffness matrices, is preserved, which otherwise would have been lost in first-order form of the system. The symmetry of matrices is especially beneficial in stability analysis of uncertain systems. In addition, it is computationally efficient as the dimension of system is lower than that of the first-order form, and sparsity and any other special nature of the original matrices are preserved which is useful in analysis and design. Stability measures of second-order systems, however, have been relatively scarce compared to those in the first-order form, even for nominal cases. A necessary and sufficient

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condition for the asymptotic stability of the nominal second-order system is the Routh–Hurwitz criterion that has been employed to develop the design procedure for stabilizing the second-order systems with variations in inertia, damping and stiffness matrices [6]. However, the Routh–Hurwitz criterion requires the knowledge of the coefficients in the characteristic polynomial of the system and the evaluation of certain determinants, which may be rather difficult to apply when the order of the system is at all large. Using the Lyapunov approach, robustness bounds for the stability of second-order systems are presented for unstructured perturbations in Hsu and Wu [7] and for dependent parametric perturbations in Cao and Shu [8]. The Lyapunov approach, however, involves solving a 2nth-order Lyapunov matrix equation for a choosing positive matrix. As a result, alternative methods such as those which provide simpler conditions directly in terms of the coefficient matrices prove to be more attractive.

For a typical second-order system with positive definite mass and stiffness matrices, it has been proved that the equilibrium is asymptotically stable if the damping matrix is also positive definite [9]. In 1997, Cox and Moro [10] studied the stability of a class of nonlinear dynamic systems whose linear part is almost classically damped and proposed a stability criterion that bounds the degree of the uncertain nonlinearity and deviation from classical damping. Although the Rayleigh damping models or other classical damping strategies are commonly used in the stability analysis due to their simplicity, they may not generally apply to real structures. Recently, Cao et al. [11] investigated the problem of the stability robustness of nonclassically damped systems with nonlinear uncertainties. In Ref. [11], using a specific Lyapunov function, bounds on nonlinear perturbations that maintain the stability of an asymptotically stable system with nonclassical damping are derived and directly expressed in terms of plant matrices.

This paper deals with the problem of the stability robustness of nonclassically damped systems with multi-directional perturbations. First of all, a simple algebraic criterion for the stability of a class of second-order system with symmetric mass, damping and stiffness matrices and multiple uncertain parameters is proposed. Secondly, based on specific Lyapunov functions, bounds on uncertain parameters that maintain the stability of an asymptotically stable, linear system with nonclassical damping are derived. The stability conditions of our criteria are directly expressed in terms of plant matrices, thus easy to check via simple algebraic computation. Since the structure information of the plant matrices are taken into consideration, the new criteria can significantly reduce the conservatism found in the literature. Moreover, the robustness bounds are not necessary symmetric with respect to the origin in the parameter space, as in the previous results [8], and this can significantly reduce the conservatism too. Finally, three simple examples are given for demonstrating the merit of the stability measures and to compare them with the existing ones.

The following notation will be used throughout this paper:

$\mathbb{R}$ ($\mathbb{C}$)	the set of all real (complex) numbers
$\mathbb{R}^n$ ($\mathbb{C}^n$)	the n-dimensional real (complex) space
$\mathbb{R}^{n\times n}$ ($\mathbb{C}^{n\times n}$)	the set of all real (complex) $n \times n$ matrices
I	the unit matrix
$\lambda_j(A)$	the jth eigenvalue of matrix A
$\lambda_{\max}(A)$ ($\lambda_{\max}(A)$)	the maximum (minimum) eigenvalue of Hermitian matrix A
A^{T}	the transpose of matrix A
$\widehat{A}$	the sum of matrix A and its transposed matrix A^{T}; $\widehat{A} = A + A^{\mathrm{T}}$
$\|x\|$	the Euclidean norm of vector x
$\det(A)$	the determinant of matrix A
$\rho(A)$	the spectral radius of matrix A
$\|A\|$	the spectral norm of matrix A; $\|A\| = \sqrt{\lambda_{\max}(A^*A)}$
$A>0$ ($A<0$)	the positive (negative) definite matrix

2. System descriptions and mathematical lemmas

A large class of dynamic systems in the field of mechanics and structures can be represented by a vector second-order differential equation of the form

$$M(q)\ddot{x}(t) + D(q)\dot{x}(t) + K(q)x(t) = 0, \tag{1}$$

where $x \in \mathbb{R}^n$ is the configuration vector, $q \in \mathbb{R}^m$ is a parametric vector, M, D and $K \in \mathbb{R}^{n\times n}$ are the mass, damping and stiffness matrices, respectively. In general, the parameters of the system, which are available for analysis, appear in the mass, the damping and the stiffness matrices. Therefore, a very important property assumed here is that the second-order differential (1) is affine in these parameters. By this affine representation we mean that

$$M(q) = M_0 + \sum_{i=1}^{m} q_i M_i, \quad D(q) = D_0 + \sum_{i=1}^{m} q_i D_i,$$
$$K(q) = K_0 + \sum_{i=1}^{m} q_i K_i, \tag{2}$$

where the nominal mass, damping and stiffness matrices M_0, D_0 and K_0 are assumed to be symmetric positive definite; q_i ($i = 1, 2, \ldots, m$) are real uncertain parameters; and M_i, D_i and K_i ($i = 1, 2, \ldots, m$) are given real constant matrices.

The following lemmas are cited and will be used in the proof of our main results.

Lemma 1 (*Anderson and Bitmead [9]*). *The system described in* (1) *is asymptotically stable if the mass, damping and stiffness matrices are symmetric positive definite, i.e.*,

$$M>0, \quad D>0, \quad K>0. \tag{3}$$

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