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On the hollowness ratio effect on the dynamics of a spinning Rayleigh beam

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Abstract

The analytical solutions of a spinning Rayleigh beam with rotatory moment inertia and gyroscopic effect are presented in this paper. The critical speeds can be written analytically in a function of the length-to-radius ratio (l) defined by the beam's length over its outer radius and the hollowness ratio (α) defined by the hollow area over the total area of the cross section. The sensitivity analyses show that the critical speed is decreasing with l, but increasing with α. Moreover, α is more sensitive to the critical speeds. The design of a spinning beam should therefore be emphasized more on the hollowness factor. Contrary to common belief, only finite critical speeds exist and the number is independent of the boundary conditions. It increases monotonically with l, but decreases with α. The steady state unbalanced response can therefore be expressed analytically by the finite precessional modes and the corresponding generalized coordinates.

Keywords: Length-to-radius ratio; Hollowness ratio; Whirl speed; Critical speed; Unbalanced response

1. Introduction

The dynamics of a rotor system is of fundamental importance in the design of rotating machinery such as stream and gas turbines, turbogenerators, internal combustion engines, turbojet engines, reciprocating and centrifugal compressors. In order to prevent from excessive vibration at resonance, analysis of whirl speed, critical speed, mode shape, and unbalanced response become very critical. Many methods for analyzing rotor systems with different parameter effects have been developed during the past years, e.g. the gyroscopic effect on the critical speeds [1], unbalanced responses due to mass eccentricity [2], disk skew [3] and residual shaft bow [4]. These methods can be divided into two categories: the discrete model by a number degrees of freedom [5–7] and the distributed parameter model, those the latter is usually difficult, if not impossible. However, in order to enhance the vibration behavior of the rotor systems shaft and reduce its weight, the accurate analytical predictions are still required.

Despite that there are hundreds of researches in rotor systems, there remain surprisingly few reports on the analytical solutions of a spinning beam. Dimentberg [8] was among the first to derive the characteristic equation of a simply supported rotating shaft. Kane [9] analyzed the whirl motion of an elastic shaft attached with a rigid disk. Eshleman and Eubanks [10] studied the critical speed of a rotor with long and short bearings. Bauer [11] conducted the relationship between natural frequencies of a spinning classical beam and a stationary one. Dynamics of a spinning beam in Timoshenko model was analyzed by Katz et al. [12]. The natural frequencies and mode shapes of a spinning Timoshekno beam in classical six boundary conditions were studied by Zu and Han [13,14]. Shiau and Hwang [15] proposed an explicit solution for the whirl speeds of a spinning Rayleigh beam in simply-supported boundary condition. Song et al. [16,17] and Librescu and Song [18] analyzed the vibration response and the flutter instability of a spinning beam modeled as a thin-walled composite beam with the effects of transverse shear, anisotropy of materials, rotatory moment inertia, and axial force loading. In the above studies, however, the number of the critical speeds and the corresponding

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Nomenclature

A	the cross-sectional area
E	Young's modulus
$e(x)$	eccentricity distribution
I_D, I_P	the diametrical and polar moment of inertia
k_D	the radius of gyration about diametrical axis
L	length of beam
l	length-to-radius ratio, $l = L/r_0$
N_m	generalized unbalanced force
n_{cr}	the number of critical speeds
$q(\tau)$	generalized coordinate
r_0,r_i	the outer and inner radii of the hollow shaft
$\bar{U}(x,t)$	complex variable, $\bar{U}(x,t) = V(x,t) + \mathrm{i}W(x,t)$
$U(\zeta,\tau)$	nondimensional variable of $\bar{U}(x,t)$
(V,W)	transverse deflections
x	axial location along the beam
(B,Γ)	rotational displacements
α	hollowness ratio, $\alpha = (r_i/r_0)^2$
λ	whirl ratio, $\lambda = \Omega/\omega$
$\varepsilon(\zeta)$	nondimensional eccentricity distribution
ρ	mass density
τ	nondimensional time
ζ	nondimensional axial location
$\phi(x)$	the associated phase angle of eccentricity distribution
$\psi_n(\zeta)$	the nth mode shape function
ω	whirl speed
$\pm\omega$	the nth forward and backward whirl speed for a given spinning speed
$\bar{\Omega}$	spinning speed
Ω	nondimensional spinning speed
Ω_n^c	the nth critical speed at resonance

forward precessional modes were believed to be infinite. Sheu and Yang [19] have recently shown that there are only finite critical speeds for a spinning Rayleigh beam in general boundary conditions. They then developed a controller design combining the finite modes and the velocity feedback to determine the optimal sensor/actuator locations and feedback gains to minimize the steady state unbalanced response [20,21].

A hollow beam with higher diametrical and polar moment of inertia per unit mass can be used to tailor the dynamics of a spinning beam. However, the analysis of the hollowness ratio effect has not been reported as yet. In this paper, the analytical solutions and the parameter sensitivity analysis of the length-to-radius ratio and the hollowness ratio for a spinning Rayleigh beam with rotatory moment inertia and gyroscopic effect are presented.

2. System modeling

The equations of motion of an undamped spinning Rayleigh beam as shown in Fig. 1 can be derived by using Hamilton's principle as [19]

$$\rho A\ddot{V} - \rho I_D \ddot{V}'' - \rho\bar{\Omega} I_D \dot{W}'' + EI_D V^{(4)} = e(x)\rho A\bar{\Omega}^2\cos(\Omega t + \phi(x)),$$
$$\rho A\ddot{W} - \rho I_D \ddot{W}'' - 2\rho\bar{\Omega} I_D \dot{V}'' + EI_D W^{(4)} = e(x)\rho A\bar{\Omega}^2\sin(\Omega t + \phi(x)), \quad (1)$$

where $2\rho\bar{\Omega}I_D\dot{W}''$ and $2\rho\bar{\Omega}I_D\dot{V}''$ are the gyroscopic effects. The equation can be rewritten in terms of a complex variable $\bar{U}(x,t)$, $\bar{U}(x,t) = V(x,t) + \mathrm{i}W(x,t)$,

$$\rho A\ddot{\bar{U}} - \rho I_D\ddot{\bar{U}}'' + \mathrm{i}2\rho\bar{\Omega}I_D\dot{\bar{U}}'' + EI_D\bar{U}^{(4)} = e(x)\rho A\bar{\Omega}^2\,\mathrm{e}^{\mathrm{i}[\bar{\Omega}t+\phi(x)]}. \quad (2)$$

For a hollow beam with circular cross section,

$$I_P = 2I_D,\ I_D = \frac{1}{4}\pi(r_0^4 - r_i^4),\ \text{and}\ A = \pi(r_0^2 - r_i^2), \quad (3)$$

where r_0 and r_i are the outer and inner radii of the hollow section, respectively. The radius of gyration (k_D) about diametrical axis can be derived as

$$k_D = \sqrt{\frac{I_D}{A}} = \frac{1}{2}r_0\sqrt{1+\alpha}, \quad (4)$$

where α is the *hollowness ratio* defined by the hollow area over the total area of the cross section as $\alpha = (r_i/r_0)^2$.

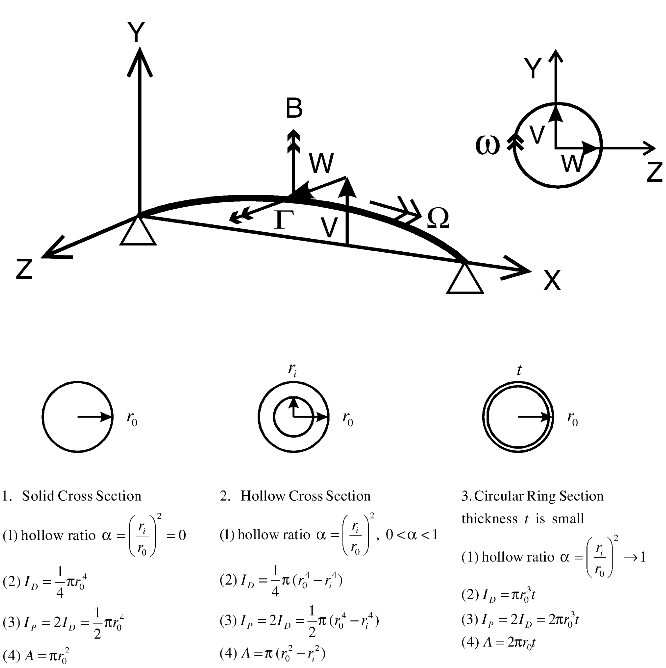

Fig. 1. Schematic diagram of a spinning hollow beam in hinged–hinged boundary conditions and the properties of the circular hollow cross sections.