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Accurate buckling loads of thin rectangular plates under parabolic edge compressions by the differential quadrature method

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Abstract

The buckling of thin rectangular plates with nonlinearly distributed loadings along two opposite plate edges is analyzed by using the differential quadrature (DQ) method. The problem is considerably more complicated since it requires that first the plane elasticity problem be solved to obtain the distribution of in-plane stresses, and then the buckling problem be solved. Thus, very few analytical solutions (the only one available in the literature is for rectangular plates with all edges simply supported) have been available in the literature thus far. Detailed formulations and solution procedures are given herein. Nine combinations of boundary conditions and various aspect ratios are considered. Comparisons are made with a few existing analytical and/or finite element data. It has been found that a fast convergent rate can be achieved by the DQ method with non-uniform grids and very accurate results are obtained for the first time. It has also been found that the DQ results, verified by the finite element method with NASTRAN, are not quite close to the newly reported analytical solution. A possible reason is given to explain the difference.

Keywords: Buckling; Differential quadrature method; Nonlinear distributed load; Rectangular plate

1. Introduction

The buckling problem of a thin rectangular elastic plate subjected to in-plane compressive and/or shear loading is important in the aircraft, civil and ship-building industries. There have been very few previous solutions for the case of nonlinearly distributed edge loadings. Perhaps this scarcity is due to the additional complexity of having to first solve for the internal pre-stress distribution as a problem in plane-stress elasticity, and then solve for the buckling problem [1]. The first work in this area was perhaps due to Van der Neut [2] in 1958, which considered a uniaxial compressive loading with a half sine distribution. Later, Benoy [3] considered a uniaxial compressive loading with a parabolic distribution and obtained an energy solution . It was pointed out by Bert and Devarakonda [1] that the works of Van der Neut [2] and Benoy [3] suffered from some serious deficiencies, such as: the distribution of the x-direction in-plane normal stress (σ_x) was assumed to depend only on the y coordinate; and the contributions of the y-direction in-plane normal stress (σ_y) and the in-plane shear stress (τ_{xy}) were ignored. Actually there is a stress-diffusion phenomenon that causes all three in-plane stress distributions to vary with x as well as with y. Recently, Bert and Devarakonda [1] removed these deficiencies and thus yielded a more accurate buckling load for the case of a thin rectangular plate with all boundaries simply supported under sinusoidal edge loadings.

Due to the complicated mathematical structure of the other boundary conditions, obtaining closed-form solutions for other combinations of boundary conditions is generally difficult. Therefore, approximate continuum or numerical methods must be resorted to for solutions. There are many such methods available, such as Rayleigh–Ritz method, finite element method, finite difference method, and Fourier series method. The differential quadrature (DQ) method, introduced by Bellman and Casti [4] in 1971, is an efficient numerical technique for the solution of initial and boundary value problems. Since Bert et al. [5] first used

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the method to solve problems in structural mechanics in 1988, the method has been applied successfully to a variety of problems [6,7].

It was found [8] that solutions by the DQ method were very sensitive to grid spacing when it was used for solving buckling problems of anisotropic rectangular plates even under uniform edge loadings. Thus, non-uniform grid spacing [9] and new ways to apply for the boundary conditions [10–12] were proposed. Accurate buckling loads of anisotropic plates under uniform or linearly distributed edge compressive loadings were obtained by DQ method [13,14]. Since pure stress boundary conditions are considered, therefore, instead of solving the second-order partial differential equations in terms of displacements, the fourth-order partial differential equation in terms of Airy stress functions, the compatibility equation, is solved by the DQ method for obtaining the in-plane stress distributions.

In view of the fact that very few previous solutions are available for the case of nonlinearly distributed edge loadings and that the DQ method and its equivalents have only been successfully used to obtain buckling loads for the cases of uniform or linearly distributed loadings, the DQ method is extended to analyze the buckling problems of thin rectangular plates subjected to parabolic distributed in-plane loadings. Formulations and procedures are given. The buckling loads for rectangular plates with nine combinations of boundary conditions and various aspect ratios are obtained and compared with available data. It has been found that a fast convergent rate can be achieved by the DQ method with non-uniform grids and very accurate results can be obtained. It has been also found that the DQ results, verified by the finite element method with NASTRAN, are not quite close to the newly reported analytical solution. A possible reason is given to explain the difference. Some conclusions are drawn based on the results reported herein.

2. Governing differential equations

Consider first the problem of in-plane elasticity, where an isotropic thin rectangular plate with side lengths of a and b, shown in Fig. 1 is under uniaxial parabolic distributed in-plane compressions. Methods based on stress functions are to be used for obtaining in-plane stresses, since all boundary conditions are in terms of stresses. The well-known Airy stress function (φ) without body forces, satisfying the governing differential equations automatically, are as follows:

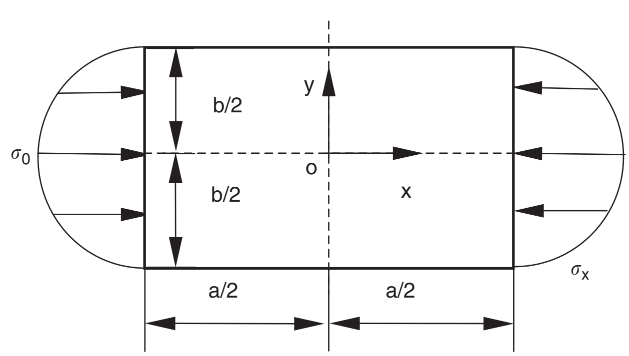

Fig. 1. Rectangular plates under parabolic distributed edge compressions.

$$\sigma_x = \frac{\partial^2\varphi}{\partial y^2}; \quad \sigma_y = \frac{\partial^2\varphi}{\partial x^2}; \quad \tau_{xy} = -\frac{\partial^2\varphi}{\partial x\partial y}. \tag{1a−c}$$

The Airy stress function should satisfy the following compatibility differential equation,

$$\frac{\partial^4\varphi}{\partial x^4} + 2\frac{\partial^4\varphi}{\partial x^2\partial y^2} + \frac{\partial^4\varphi}{\partial y^4} = 0. \tag{2}$$

Once φ is obtained, the in-plane stresses σ_x, σ_y, τ_{xy} can be computed by Eq. (1). To obtain solution φ numerically by the DQ method, appropriate boundary conditions should be applied. It should be emphasized that one cannot apply the stress boundary conditions in terms of differential quadrature directly. Let $\bar{X}$ and $\bar{Y}$ are the known in-plane load components acting on the boundaries in the x and y directions; then φ and its first partial derivatives along boundaries can be computed by

$$\begin{aligned} \varphi_B &= \int_A^B (y_B - y)\bar{X}\,ds - \int_A^B (x_B - x)\bar{Y}\,ds, \\ (\varphi_x)_B &= \left(\frac{\partial\varphi}{\partial x}\right)_B = -\int_A^B \bar{Y}\,ds, \\ \left(\varphi_y\right)_B &= \left(\frac{\partial\varphi}{\partial y}\right)_B = \int_A^B \bar{X}\,ds, \end{aligned} \tag{3a−c}$$

where A and B are two arbitrary distinguished points on the boundary. Since superimposing a linear function to Airy stress function (φ) will not affect the stress values, one can assume that

$$\varphi_A = 0; \quad \left(\frac{\partial\varphi}{\partial x}\right)_A = 0; \quad \left(\frac{\partial\varphi}{\partial y}\right)_A = 0. \tag{4a−c}$$

One can choose point 1 shown in Fig. 2 as point A. For the loading to be considered in this paper, shown in Fig. 1, $\bar{Y} = 0$ and the only non-zero $\bar{X}$ along the plate boundary is:

$$\begin{aligned} \bar{X}\left(x = \frac{a}{2}\right) &= \frac{4\sigma_0}{b^2}\left(y^2 - \frac{b^2}{4}\right), \\ \bar{X}\left(x = -\frac{a}{2}\right) &= -\frac{4\sigma_0}{b^2}\left(y^2 - \frac{b^2}{4}\right). \end{aligned} \tag{5a, b}$$

Substituting Eq. (5) into (3) and using Eq. (4) yield

$$(1) \quad y = -\frac{b}{2}, \qquad \varphi_i = \left(\frac{\partial\varphi}{\partial y}\right)_i = 0, \tag{6a}$$

$$\begin{aligned} (2) \quad x = \frac{a}{2}, \quad \varphi_i &= \frac{\sigma_0}{48b^2}\left(16y_i^4 - 3b^4 - 16b^3y_i - 24b^2y_i^2\right), \\ \left(\frac{\partial\varphi}{\partial x}\right)_i &= 0, \end{aligned} \tag{6b}$$