International Journal of Plasticity 53 (2014) 228–240

Contents lists available at ScienceDirect

International Journal of Plasticity

journal homepage: www.elsevier.com/locate/ijplas

An explicit, straightforward approach to modeling SMA pseudoelastic hysteresis

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ARTICLE INFO

Article history:
Received 30 April 2013
Received in final revised form 12 August 2013
Available online 24 August 2013

Keywords:
Large deformations
Pseudoelasticity
J_2-flow models
Nonlinear combined hardening
Explicit approach

ABSTRACT

Toward a straightforward approach to modeling SMA pseudoelastic behavior, a new finite J_2-flow elastoplastic model with nonlinear combined hardening is proposed by coupling the size change of the yield surface with the moving of the yield surface center. A departure from usual approaches is that any given shapes of hysteresis loops may be automatically generated via explicit procedures in the framework of classical J_2 elastoplasticity with no reference to any phase variables. Numerical examples are presented to show good accord with test data for Ti–Ni alloys.

1. Introduction

In the past decades, investigations in modeling remarkable SMA thermomechanical behavior, in particular, pseudoelastic hysteresis and shape memory effects etc., have been carried out based on the microstructural mechanisms of solid–solid transitions between martensites and austenites. Numerous results have been derived from micoscopic, mesoscopic (micro–macro), and phenomenological (macroscopic) standpoints, separately. Microscopic models are mainly concerned with microscopic features such as nucleation and twin growth etc. and instrumental in understanding fundamental phenomena occurring at the microscopic level. On the other hand, mesoscopic or micro–macro studies derive constitutive relations by establishing relationships between micromechanics and continuum mechanics. Both provide significant predictions and results, but do not appear to be appropriate for the purpose of designing engineering structures and devices. For main contributions prior to 2006 in these two respects, reference may be made to the extensive review articles by Patoor et al. (2006) and Lagoudas et al. (2006), as well as relevant monographs, e.g., Auricchio (1995) and Lagoudas (2008).

In contrast, phenomenological or macroscopic models are based on macroscopic variables directly related to engineering designs and analyses and thus suitable for engineering applications. Here we focus on such models. Literature in this respect seems immense. Below are some representative samples.

Earlier, Falk (1980) proposed a polynomial potential model with a Landau–Devonshire free-energy function. Such models are of simple form and apply to simple cases. Huo (1989) made a further study of SMA pseudoelastic hysteresis based also on a Landau–Devonshire free-energy with four parameters. Brocca et al. (2002) presented a microplane model by developing the slip theory for polycrystal plasticity.

Internal variables for phase changes were not introduced in the above studies. A widely used approach is based on introduction of suitable internal variables in an averaged sense of representing microstructural features relating to the martensite-austenite phase transitions. It appears that the first application of this approach was made by Tanaka and Nagaki (1982).

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0749-6419/$ - see front matter
http://dx.doi.org/10.1016/j.ijplas.2013.08.010

Subsequent developments for small strain cases may be found in the aforementioned review articles and monographs and the references therein. Here, for contributions prior to 2006, only certain representative samples treating SMA pseudoelastic behavior at finite strain are mentioned, including Tanaka et al. (1986), Masud et al. (1997), Auricchio (2001), Helm and Haupt (2003). In addition, newest contributions and developments in multi-axial modeling may be found in, e.g., Feng and Sun (2007), Popov and Lagoudas (2007), Peng et al. (2008), Arghavani et al. (2010a), Morin et al. (2011), Saleeb et al. (2011), Yu et al. (2013) for small strain cases, and Müller and Bruhns (2006), Helm (2007), Ziólkowski (2007), Luig and Bruhns (2008), Thamburaja (2010), Arghavani et al. (2011) and Teeriaho (2013) for finite strain cases. Moreover, results treating martensite reorientation are derived by Panico and Brinson (2007), Arghavani et al. (2010b), Zaki (2012) and Auricchio et al. (2007) obtained results for cases coupled with permanent inelasticity. Results with numerical treatments are presented by Moumni et al. (2008), Reese and Christ (2008), Stein and Sager (2008), Hartl et al. (2010), Arghavani et al. (2011) and Lagoudas et al. (2012). In particular, reference may be made to a collaborative study by Sittner et al. (2009) for comparisons and validations of representative models with extensive test data.

Motivated by the idea of applying elastoplasticity models in various extended senses (see, e.g., Bertram, 1982; Graesser and Cozzarelli, 1994; Delobelle and Lexcellent, 1996; Lubliner and Auricchio, 1996; Trochu and Qian, 1997) as well as the findings of recoverable elastoplastic flows within the very framework of classical elastoplasticity in a serial study (cf. Xiao et al., 2010a,b, 2011), in this contribution an explicit, straightforward approach from a phenomenological standpoint, which represents a substantial extension of the previous study (Xiao, 2012), is suggested to treat pseudoelastic hysteresis in a broad sense. It will be shown that a new, substantial development in treating nonlinear combined hardening should be made toward modeling the pseudoelastic hysteresis in a general case. It is found that this development will lead to an essential coupling between the size change of the yield surface and the moving of the yield surface center, which has not been treated in literature. A new hardening equation of this nature will be proposed.

With the above idea, it is shown that combined hardening J_2 elastoplasticity models with substantial coupling may be used to accurately simulate pseudoelastic hysteresis loops of any given shapes via direct, explicit procedures. This article will be organized with the main contents as follows: In Section 2 we introduce combined hardening J_2-flow model with substantial coupling in a thermodynamically consistent sense; in Section 3 we present explicit expressions of nonlinear hardening functions for the purpose of automatically generating hysteresis loops of any given shapes; in Section 5 we take flag-like loops for SMAs into account and establish a direct, explicit approach to matching test data; in Section 5 we present numerical examples to show good accord with test data; and finally we discuss the approach proposed and the main results din Section 6.

2. Combined hardening J_2-flow models with substantial coupling

As indicated in the introduction, a departure from usual elastoplastic models is that we need to treat a general case of nonlinear combined hardening with a yield strength relying both on the plastic work and on the back stress, thus coupling the changing of the size of the yield surface with the moving of the yield surface center. The main purpose of this section is to establish new J_2-flow models of this property.

Various formulations have been suggested for finite elastoplastic deformations. Of them, a natural, direct extension of the classical Prandtl–Reuss theory at small strain to finite strain leads to Eulerian formulation and has been widely used. Its consistent formulation has been established in recent years (cf., e.g. Xiao et al., 2006, 2007). In what follows, we shall direct attention to this consistent Eulerian formulation.

2.1. Nonlinear combined hardening with substantial coupling

Let $\boldsymbol{F}$, $\boldsymbol{D}$ and $\boldsymbol{W}$ be the deformation gradient, the stretching and the vorticity spin. The latter two are the symmetric and anti-symmetric parts of the velocity gradient $\dot{\boldsymbol{F}} \cdot \boldsymbol{F}^{-1}$. Moreover, let $\boldsymbol{\tau}$ be the Kirchhoff stress, i.e. the Cauchy stress $\boldsymbol{\sigma}$ weighted by the deformation Jacobian $J = \det \boldsymbol{F}$ and then $\boldsymbol{\tau} = J\boldsymbol{\sigma}$. Its deviatoric part is denoted $\tilde{\boldsymbol{\tau}}$. Throughout, the superimposed dot means the time derivative.

For small deformations, the stretching $\boldsymbol{D}$ and the Kirchhoff stress $\boldsymbol{\tau}$ may be reduced to the infinitesimal strain rate $\dot{\boldsymbol{\varepsilon}}$ and the Cauchy stress $\boldsymbol{\sigma}$, i.e.

$$\boldsymbol{D} \approx \dot{\boldsymbol{\varepsilon}}, \quad \boldsymbol{\tau} \approx \boldsymbol{\sigma},$$

at small deformations. The starting point of the classical Prandtl–Reuss theory is the decomposition (cf., e.g., Khan and Huang, 1995):

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^p.$$

The elastic part $\dot{\boldsymbol{\varepsilon}}^e$ is given by an equivalent rate form of Hooke's law, *viz.*

$$\dot{\boldsymbol{\varepsilon}}^e = \frac{1}{2G}\dot{\boldsymbol{\sigma}} - \frac{\nu}{E}(\mathrm{tr}\dot{\boldsymbol{\sigma}})\boldsymbol{I},$$

while the plastic part $\dot{\boldsymbol{\varepsilon}}^p$ may be governed by a normality flow rule, namely,