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Effective elastic constants of two-dimensional cellular materials with deep and thick cell walls

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Abstract

In order to analyse the elastic constants of cellular materials with deep and thick cell walls, finite element analysis using two kinds of unit cell approach (stiffness matrix method and compliance matrix method) is performed which is applicable to any orthotropic cellular materials. Comparison between results from the FEA, the theories presented in this paper and experiments of previous investigators indicate that the elastic constants of cellular materials with thick cell walls depend not only on the relative density but also on the joint stiffening effect. Approximate formulae under generalised plane strain conditions are also presented for the purpose of obtaining the effective elastic constants for cellular materials with deep and thick cell walls. A satisfactory agreement was found with experimental results obtained on a deep and thick cellular material. The results indicate that the previous models in which the wall of cellular materials is treated as a simple beam are not adequate to evaluate the effective elastic constants of cellular materials with deep and thick cell walls. In addition, considerable attention needs for the measurement of effective Young's modulus of square cellular materials in the two soft directions because it is strongly affected by misalignment errors.

Keywords: Cellular materials; Deep and thick cell walls; Elastic constants; Joint stiffening effect; Generalised plane strain

1. Introduction

During the last 10 years, there has been extensive modelling of cellular materials. Much work has been performed on two-dimensional (2D) and three-dimensional (3D) structures. Some of

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Nomenclature

$A(x)$	area function of cell walls
A_J	joint area of cell walls
b	cell wall depth of cellular materials
C_b, C_a, C_s	shear modulus components
E^*	effective Young's modulus of cellular materials
$(E^*)_{stiff}$	effective Young's modulus of square cellular materials in two stiff directions
E	Young's modulus of solid material
G^*	effective shear modulus of cellular materials
$(G^*)_{stiff}$	effective shear modulus of square cellular materials in two stiff directions
G	shear modulus of solid material
h, l	cell wall lengths of cellular materials
$I(x)$	second moment of area of cell walls
j	joint stiffening factor
k	shear correction coefficient
M_h	axial compliance of the cell wall with length h
M_l	axial compliance of the cell wall with length l
N_h	bending compliance of the cell wall with length h
N_l	bending compliance of the cell wall with length l
v^*	effective Poisson's ratio of cellular materials
$(v^*)_{stiff}$	effective Poisson's ratio of square cellular materials in two stiff directions
v	Poisson's ratio of solid material
θ	angle between the horizontal and the cell wall with length l (in-plane)
φ	rotation angle of cellular model from its reference frame
ρ_M^*	relative density of cellular materials

these models used computer models with multiple cells, and included various imperfections and irregularities [1–7]. The mechanical behaviour of cellular materials can be modelled by idealising the observed structures, characterising the cell wall properties, and analysing the mechanisms by which the cell wall deforms [8–15]. Kim and Al-Hassani [14,15] recently described the mechanical properties of general hexagonal cellular materials comprised of doubly tapered struts. In the present paper, this type of analysis is applied to the 2D cellular materials with deep and thick cell walls.

Polymer and ceramic cellular materials used for bone replacement scaffolds and catalyst supports sometimes have square or triangular cellular structures, either to increase the surface area or to give a stiffer structure [16–18]. The depth of these cellular materials is frequently large relative to the cell wall length and the cell walls are thick, so they cannot be approximated by either a thin plate or a slender beam. Thus, the effective elastic constants derived from an analysis in which the cell wall is treated as a beam are not realistic for such cellular materials.

Gulati [17] measured the effective elastic constants for square cellular materials by using sonic resonance and deflection techniques and compared theoretical results with experimental values. The experimental values of effective Young's moduli were higher than those predicted by his theory.