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Simulation of distortional hardening by generalizing a uniaxial model of finite strain viscoplasticity

M. Freund *, A.V. Shutov, J. Ihlemann

Chemnitz University of Technology, Professorship of Solid Mechanics, Strasse der Nationen 62, 09111 Chemnitz, Germany

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ABSTRACT

In this paper we generalize a uniaxial model of finite strain viscoplasticity (proposed by Shutov and Kreißig) using the so-called concept of representative directions. As a result, a new three-dimensional phenomenological material model is obtained. The original model takes the nonlinear isotropic and kinematic hardening into account, but it does not cover the distortional hardening. Using a series of numerical computations we show that the isotropic and kinematic hardening is completely retained during the process of generalization. Moreover, the distortional hardening effects are automatically induced by the concept itself. This is demonstrated by simulating combined tension–torsion tests on thin-walled tubular specimens. Furthermore, the generalized material model is validated by a comparison with real experimental data concerning the shape of the yield surface. A good correspondence between the simulation results and the measurements is observed.

1. Introduction

For the general introduction to the theory of viscoplasticity see, for example, Lemaitre and Chaboche (1990) or Haupt (2002). The numerical treatment of the corresponding constitutive equations is discussed in the monograph of Simo and Hughes (1998).

Even after a small amount of inelastic deformation, the plastic properties of metallic materials may exhibit a strong anisotropy (Khan et al., 2009, 2010). In particular, the Bauschinger effect is observed under reverse loading. The common approach to the phenomenological modeling of the Bauschinger effect was suggested by Prager (1935) by the introduction of the kinematic hardening concept. This concept was then generalized by Armstrong and Frederick (1966) in order to take the saturation of the kinematic hardening into account. An important feature of this concept is the implementation of back stresses (micro stresses) which reflect the plastic anisotropy accumulated in the material.[1,2] Some studies do not deal with the evolution of the yield surface but rather concentrate on the description of its initial shape (see Barlat et al., 2005). Such simplified models can be used in applications with very small inelastic deformations. Better predictive capabilities can be achieved if, additionally to the distorted initial yield surface, nonlinear isotropic and kinematic hardening are introduced (see Wu, 2002). For such approaches, the form of the yield surface remains constant.

The classical material model of Chaboche and Rousselier (1983) contains the concept of Armstrong and Frederick. This model has the advantage that it admits a simple rheological interpretation in form of a Schwedoff-body (Fig. 2(a)). Several strategies can be adopted for the generalization of this model to finite strains (see, for example, Dogui and Sidoroff, 1985; Tsakmakis, 1996; Lührs et al., 1997; Svendsen et al., 1998; Mollica et al., 2001). Some of the generalizations are based on

* Corresponding author. Tel.: +49 371 531 38673; fax: +49 371 531 838673.
E-mail address: michael.freund@mb.tu-chemnitz.de (M. Freund).

[1] A critical review of this concept is presented by Kafka and Vokoun (2005).

[2] A micromechanical approach to the modelling of the Bauschinger effect is presented by Rauch et al. (2007).

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the double multiplicative split of the deformation gradient proposed by Lion (2000). This elegant assumption is directly motivated by the rheological model of the Schwedoff-body mentioned above. Material models which implement the double multiplicative split were presented by Helm (2001), Tsakmakis and Willuweit (2004), Dettmer and Reese (2004), Shutov and Kreißig (2008a), Vladimirov et al. (2008) as well as Henann and Anand (2009).

In the case of nonproportional loading conditions with abrupt change of the loading path, the distortion of the shape of the yield surface must be taken into account. The approach of polycrystalline metal plasticity allows to capture the distortion of the yield surface even at finite strains (see Schurig et al., 2007; Wang et al., 2008; Rousselier et al., 2009; Fang et al., 2011). Those models can be motivated by the consideration of microstructure evolution such as activation of slip systems, reorganization of dislocation structures, cross-hardening effects and so on. But, due to the large CPU-times, the models of polycrystalline plasticity are not optimal for large-scale FEM computations.[3] Instead, simplified phenomenological models are usually used.

Unfortunately, in the field of finite strain metal plasticity there exist only a few phenomenological models that are able to capture the distortional hardening. A material model which is based on the corotational rates of the logarithm of the left stretch tensor was proposed by Yeganeh (2007). The evolution of the Hill-type anisotropy is captured using the plastic strain induced anisotropy tensor. A finite strain viscoplasticity model was proposed by Böhlke et al. (2008), which takes the crystallite orientation distribution into account. Bucher et al. (2004), basing on the ideas of Mandel (1973) and Dafalias (1987), introduced a new substructure intermediate configuration. The model captures hardening stagnation and the cross-hardening effect. Finally, we mention a phenomenological model of finite strain viscoplasticity proposed by Shutov et al. (2011), which can be motivated by a two-dimensional rheological model. The rheological model can be considered as a generalization of the one-dimensional Schwedoff body, which is achieved by the introduction of a new idealized rheological element. An important feature of this model is that only second-rank backstress-like tensors are used in order to capture the distortion of the yield surface.

In the present paper a uniaxial formulation of a model of finite strain viscoplasticity (Shutov and Kreißig, 2008a) is generalized to a new three-dimensional constitutive model with the concept of representative directions (Freund and Ihlemann, 2010). Note that the original model of Shutov and Kreißig (2008a) does not allow to take the change of the shape of the yield surface into account. Even more, the phenomena of distortional hardening remain unexposed under uniaxial tension and compression. Therefore, it is an interesting result that the combination of this model with the concept of representative directions *naturally yields a model with distortional hardening*. In our opinion, this is the main benefit of the proposed approach, since it would cost a lot of work to incorporate this effect into the original three-dimensional model.

We conclude this introduction with a few words regarding notation. In this paper, a coordinate-free tensor setting is used (see, for example Itskov, 2007; Shutov and Kreißig, 2008b). The rank of the tensor is indicated by the number of underlines. The deviatoric part of a second rank tensor $\underline{\underline{X}}$ is defined by $(\underline{\underline{X}})' = \underline{\underline{X}} - \frac{1}{3}\mathrm{tr}(\underline{\underline{X}})\ \underline{\underline{I}}$ with $\underline{\underline{I}}$ as the identity tensor. For the dyadic product we use the symbol "$\circ$". Finally, the Lagrangian time derivative of a second-rank tensor is denoted by $\overset{\triangle}{\underline{\underline{X}}}$.

2. The concept of representative directions

The concept of representative directions is intended to generalize one-dimensional material models to fully three-dimensional constitutive models for the use within the finite element method. It was already described in Freund and Ihlemann (2010) and Freund et al. (2011). The first step of this generalization algorithm is to use the deformation gradient $\underline{\underline{F}}$ to compute the right Cauchy–Green tensor

$$\underline{\underline{C}} = \underline{\underline{F}}^T \cdot \underline{\underline{F}} \tag{1}$$

as a proper measure of the pure state of strain at a material point. Next, the isochoric part $\overset{G}{\underline{\underline{C}}}$ of the right Cauchy–Green tensor is derived by using the volume ratio J_3 which is the determinant of the deformation gradient $\underline{\underline{F}}$.

$$\overset{G}{\underline{\underline{C}}} = J_3^{-\frac{2}{3}} \underline{\underline{C}} \quad \text{with} \quad J_3 = \frac{\mathrm{d}V}{\mathrm{d}\widetilde{V}} = \det \underline{\underline{F}} \tag{2}$$

Due to this isochoric deformation, the stretch $\overset{\alpha}{\lambda}$ of any material fixed line along the individual direction $\overset{\alpha}{\underline{e}}$ concerning the reference configuration is determined by

$$\overset{\alpha}{\lambda} = \sqrt{\overset{\alpha}{\underline{e}} \cdot \overset{G}{\underline{\underline{C}}} \cdot \overset{\alpha}{\underline{e}}} \quad \text{with} \quad \| \overset{\alpha}{\underline{e}} \| = 1. \tag{3}$$

Suppose that for each of those discrete directions $\overset{\alpha}{\underline{e}}$ (so-called representative directions) a corresponding uniaxial stress response can be computed by a one-dimensional material model. In general, the approach is applicable to inelastic material behavior. Suppose, that the uniaxial second Piola–Kirchhoff stresses $\overset{\alpha}{\widetilde{T}}$ also depend on the stretch rate $\dot{\overset{\alpha}{\lambda}}$ and some internal variables $\overset{\alpha}{\Gamma}_i$.

[3] At the same time, some simplified polycristalline modeling allows to reduce the computational costs (see Rousselier and Leclercq, 2006).