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Constitutive equation for friction with transition from static to kinetic friction and recovery of static friction

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Abstract

A high friction coefficient is first observed as a sliding between bodies commences, which is called the *static friction*. Then, the friction coefficient decreases approaching the lowest stationary value, which is called the *kinetic friction*. Thereafter, if the sliding stops for a while and then it starts again, the friction coefficient recovers and a similar behavior as that in the first sliding is reproduced. In this article the *subloading-friction model* with a smooth elastic–plastic sliding transition [Hashiguchi, K., Ozaki, S., Okayasu, T., 2005. Unconventional friction theory based on the subloading surface concept. Int. J. Solids Struct. 42, 1705–1727] is extended so as to describe the reduction from the static to kinetic friction and the recovery of the static friction. The reduction is formulated as the plastic softening due to the separations of the adhesions of surface asperities induced by the sliding and the recovery is formulated as the viscoplastic (creep) hardening due to the reconstructions of the adhesions of surface asperities during the elapse of time under a quite high actual contact pressure between edges of asperities.

Keywords: Constitutive equation; Friction; Plasticity; Rate-dependency; Subloading surface model

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1. Introduction

Description of the friction phenomenon as a constitutive equation has been attained first as a rigid-plasticity (Seguchi et al., 1974; Fredriksson, 1976). Further, it has been extended to an elastoplasticity (Michalowski and Mroz, 1978; Oden and Pires, 1983a,b; Curnier, 1984; Cheng and Kikuchi, 1985; Oden and Martines, 1986; Kikuchi and Oden, 1988; Wriggers et al., 1990; Peric and Owen, 1992; Anand, 1993; Mroz and Stupkiewicz, 1998; Gearing et al., 2001; Laulsen, 2001; Wriggers, 2003) in which the penalty concept, i.e. the elastic springs between contact surfaces is incorporated and the isotropic hardening is taken into account so as to describe the test results (cf. Courtney-Pratt and Eisner, 1957) exhibiting the smooth contact traction vs. sliding displacement curve reaching the static-friction. However, the interior of the sliding-yield surface has been assumed to be an elastic domain and thus the plastic sliding velocity due to the rate of traction inside the sliding-yield surface is not described. Therefore, the accumulation of plastic sliding due to the cyclic loading of contact traction within the sliding-yield surface cannot be described by these models. They could be called the conventional friction model in accordance with the classification of plastic constitutive models by Drucker (1988). On the other hand, the first author of the present article has proposed the *subloading surface model* (Hashiguchi, 1980, 1989; Hashiguchi and Tsutsumi, 2006) within the framework of unconventional plasticity, which is capable of describing the plastic strain rate by the rate of stress inside the yield surface. Based on the concept of subloading surface, the authors proposed the *subloading-friction model* (Hashiguchi et al., 2005; Ozaki et al., 2007), which describes the smooth transition from the elastic to plastic sliding state and the accumulation of sliding displacement during a cyclic loading of tangential contact traction. Besides, in this model the reduction of friction coefficient with the increase of normal contact traction observed in experiments (cf. e.g. Bay and Wanheim, 1976; Dunkin and Kim, 1996; Gearing et al., 2001) is formulated by incorporating the nonlinear sliding-yield surface, while the decrease has not been taken into account in Coulomb sliding-yield surface, which has been adopted widely in constitutive models for friction so far.

It is widely known that when bodies at rest begin to slide to each other, a high friction coefficient appears first, which is called the *static friction*, and then it decreases approaching a stationary value, called the *kinetic friction*. However, this process has not been formulated pertinently so far, although the increase of friction coefficient up to the peak has been described as the isotropic hardening, i.e. the expansion of sliding-yield surface as described above.

Further, it has been found that if the sliding ceases for a while and then it starts again, the friction coefficient recovers and the similar behavior as that in the initial sliding is reproduced (Dokos, 1946; Rabinowicz, 1951, 1958; Howe et al., 1955; Derjaguin et al., 1957; Brockley and Davis, 1968; Kato et al., 1972; Richardson and Noll, 1976; Horowitz and Ruina, 1989; Ferrero and Barrau, 1997; Bureau et al., 2001). The recovery has been formulated by equations including the time elapsed after the stop of sliding. However, the inclusion of time itself leads to the loss of objectivity in constitutive equations as known from the fact that the evaluation of elapsed time varies depending on the judgment of time when the sliding stops, which is accompanied with the arbitrariness especially for the state varying sliding velocity in low level. Generally speaking, the variation of material property cannot be described pertinently by the elapse from a particular time but has to be described by state of internal variables without the inclusion of time itself.