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A generic anisotropic continuum damage model integration scheme adaptable to both ductile damage and biological damage-like situations

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This paper aims at presenting a general versatile time integration scheme applicable to anisotropic damage coupled to elastoplasticity, considering any damage rate and isotropic hardening formulations. For this purpose a staggered time integration scheme in a finite strain framework is presented, together with an analytical consistent tangent operator. The only restrictive hypothesis is to work with an undamaged isotropic material, assumed here to follow a J_2 plasticity model. The only anisotropy considered is thus a damage-induced anisotropy. The possibility to couple any damage rate law with the present algorithm is illustrated with a classical ductile damage model for aluminium, and a biological damage-like application. The later proposes an original bone remodelling law coupled to trabecular bone plasticity for the simulation of orthodontic tooth movements. All the developments have been considered in the framework of the implicit non-linear finite element code Metafor (developed at the LTAS/MN^{2}L, University of Liège, Belgium – www.metafor.ltas.ulg.ac.be).

1. Introduction

Damage mechanics deals with modelling the loss of stiffness and progressive microscopic failure mechanisms induced by external loading in a material. Coupled damage models are models in which damage is incorporated into constitutive equations. Their use can lead to the development of complex constitutive equations whose numerical integration has to be considered adequately.

Two main coupled approaches to damage mechanics can be found in the literature.

The first one is a micromechanical approach to damage based on the work of Gurson (1977) who considered the growth of spherical voids in a plastic material. The extensions to the Gurson–Tvergaard–Needlemann (GTN) model accounted for plastic hardening and physically described the ductility of materials (Tvergaard and Needleman, 1984; Rousselier, 1987). GTN models define the damage variable as the void fraction, and its evolution is due to the nucleation, growth and coalescence of voids. While initially an isotropic approach to damage, it has been extended to anisotropic damage (Hammi and Horstemeyer, 2007; Zapara et al., 2012; Horstemeyer and Bammann, 2010).

The second one is a phenomenological approach to damage often referred to as the Continuum Damage Mechanics (CDM). It should be noted however that the GTN approach is also a continuum approach to damage and thus that the

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terminology CDM may be considered as improper. Phenomenological damage models based on the concept of effective stress space were introduced by Kachanov (1958) and later by Rabotnov (1968) [as cited in Voyiadjis and Kattan (2006) and Lemaitre and Desmorat (2005)] who were the first to introduce a scalar damage variable which may be interpreted as the effective surface density of micro-cracks per unit volume. The CDM is derived from a thermodynamic framework, ensuring the positivity of the dissipation. The damage variable is an internal variable related to the effective density of cracks or cavities at each point (for the isotropic case) or at each point and in each direction (anisotropic case), that is, to the micro-structure. As for the GTN approach, while the CDM was initially developed considering a scalar damage variable, it has been extended to a tensor representation describing a damage-induced anisotropy (Voyiadjis et al., 2008; Desmorat and Otin, 2008; Brünig et al., 2008; Abu Al-Rub and Voyiadjis, 2003; Badreddine et al., 2010; Brodland et al., 2006; Dunand et al., 2012).

For both approaches, a description of the coupling between damage and elasto-plasticity has to be completed with a damage evolution law. While damage evolution laws were proposed in the original works deriving the GTN or CDM approaches, several other damage rate expressions can be found in the literature describing different damage mechanisms (Duddu and Waisman, 2013; Souza and Allen, 2012; Tekoglu and Pardoen, 2010; Hammi and Horstemeyer, 2007; Qi and Bertram, 1999; Kitzig and Häußler-Combe, 2011; Lecarme et al., 2011; Khan and Liu, 2012; Lai et al., 2009; Zairi et al., 2011) and coupling with time-, rate-, and temperature-dependent materials (Abu Al Rub and Darabi, 2012; Besson, 2009; Stewart et al., 2011; Horstemeyer et al., 2000; Guo et al., 2013), to cite only a few.

In the present work, we will use the phenomenological approach of anisotropic damage, i.e. anisotropic Continuum Damage Mechanics. This choice is driven by the wide range of applications of CDM. In particular, and as will be treated further in this work, it can be extended to represent a stiffness softening not linked to the growth of micro-cracks but to other phenomena, such as a bio-chemical coupling. Considering biological effects coupled to mechanical loading, we will be interested in describing the effect of biological actions on the material behaviour rather than describing in details the biological phenomena. The use of a CDM approach in this context rather than a GTN approach of damage is thus straightforward to describe the evolution of the stiffness tensor due to external loading. Besides, the coupling between elasticity and damage in those cases plays a strong role which is naturally included into the CDM approach. In the case of GTN models, the elastic properties can also be functions of damage but this dependence is often neglected.

In this work an additive decomposition of the strain rate is assumed to model the elastoplasticity in finite strains. It should however be noted that several studies also developed mathematical frameworks to couple anisotropic damage and multiplicative elastoplasticity (Menzel et al., 2005; Brünig, 2002, 2003a; Ekh et al., 2004). This approach to elastoplasticity in large strains leads to a completely different formulation of damage (Brünig, 2003b). The computational tools developed to integrate anisotropic damage in that case are thus not applicable in the present work.

The numerical integration algorithms of constitutive models incorporating anisotropic damage effects presented in the literature (Lemaitre and Desmorat, 2005; Borgqvist and Wallin, 2013; El khaoulani and Bouchard, 2013; Brünig et al., 2008; de Souza Neto et al., 2011) are usually limited to one given damage model and are not easily extended to other formulations or damage criteria. The numerical scheme in Simo and Ju (1987a,b) or in Jeunechamps and Ponthot (2013) is similar to the proposed approach in such a way that the integration can be considered as a triple operator split: elastic predictor, plastic corrector, damage corrector. However this numerical scheme was developed for isotropic damage, i.e. with a scalar equation to solve for the damage correction and direct decoupling of damage and plasticity for the plastic correction. For those two reasons, several other generic integration schemes have been proposed in the case of isotropic damage (de Souza Neto et al., 1994; de Souza Neto and Perić, 1996; Doghri, 1995; Vaz and Owen, 2001; Mashayekhi et al., 2005; Boers et al., 2005). Simo and Ju (1987a,b) also propose a numerical scheme for the integration of anisotropic damage. However, in that case, a strain-based anisotropic damage is assumed, thus leading to a different numerical approach.

The present work is aimed at developing a generic phenomenological anisotropic damage integration scheme that can be coupled with any isotropic hardening law and damage rate. The only restrictive hypothesis was to work with a material whose undamaged behaviour can be modelled with a von-Mises elasto-plastic model. The anisotropy of the material is thus only a damage-induced anisotropy. Otherwise, any thermodynamically consistent damage criterion can be used in conjunction with the proposed original staggered integration scheme. Furthermore a finite strains assumption is considered for the anisotropic continuum damage formulation thus allowing the use of the model in large strains and large rotations applications. The developed algorithm can be also coupled to damage criteria as different as criteria describing ductile damage or biological damage-like phenomena. Considering the later case, an original, enhanced extension of a small-strain elastic-damage model is developed in this work and applied to simulate a tooth displacement in an orthodontic treatment. This work thus presents an original fully coupled non-linear model of bone remodelling occurring during orthodontic tooth movement.

Beyond this introduction, this paper is divided into three main sections. The first section presents the extension of CDM to a finite strain formulation considering an anisotropic symmetric second order damage tensor. Using a second order tensor restricts the anisotropy to orthotropy. This extension makes no assumption on the damage rate except that it remains a symmetric tensor. A new implicit time integration algorithm in a finite element context is then proposed. The following two sections demonstrate the versatility of the approach. First a ductile damage model was used to verify the proposed approach by comparison to another integration scheme from the literature (de Souza Neto et al., 2011). For this, a simple uniaxial test was reproduced to compare the present results with those of Aboudi (2011). Second, a biologically driven damage model was developed. Its aim was to propose a model of orthodontic tooth movement considering biological softening and hardening of bone tissue.