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An affine formulation for the self-consistent modeling of elasto-viscoplastic heterogeneous materials based on the translated field method

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ABSTRACT

The modeling of heterogeneous materials with an elasto-viscoplastic behavior is generally complex because of the differential nature of the local constitutive law. Indeed, the resolution of the heterogeneous problem involves space–time couplings which are generally difficult to estimate. In the present paper, a new homogenization model based on an affine linearization of the viscoplastic flow rule is proposed. First, the heterogeneous problem is written in the form of an integral equation. The purely thermoelastic and purely viscoplastic heterogeneous problems are solved independently using the self-consistent approximation. Using translated field techniques, the solutions of the above problems are combined to obtain the final self-consistent formulation. Then, some applications concerning two-phase fiber-reinforced composites and polycrystalline materials are presented. When compared to the reference solutions obtained from a FFT spectral method, a good description of the overall response of heterogeneous materials is obtained with the proposed model even when the viscoplastic flow rule is highly non-linear. Thanks to this approach, which is entirely formulated in the real-time space, the present model can be used for studying the response of heterogeneous materials submitted to complex thermo-mechanical loading paths with a good numerical efficiency.

1. Introduction

The self-consistent scheme is a common homogenization method that was developed to connect local deformation mechanisms to the overall behavior of heterogeneous disordered materials (Hershey, 1954; Kröner, 1958). In the past decades, many efforts have been made to obtain extensions of the self-consistent approximation to the non-linear case. For heterogeneous materials with a rate-independent elastoplastic behavior, Kröner (1961) proposed a model where the plastic strain is considered as a stress-free strain (i.e. eigenstrain). However, internal stresses are largely overestimated with this method which, like Taylor's model (Taylor, 1938), leads to uniform plastic strains. As an alternative, Hill (1965) developed an incremental version of the self-consistent approximation. The constitutive equations are written in the form of a pseudo-linear relation such that it is possible to use the self-consistent approximation at each stage of a loading path. Extending Hill's idea to the case of viscoplasticity, Hutchinson (1976) applied the self-consistent method by introducing a viscoplastic modulus which results from the linearization of the constitutive relation. This approach has been later modified by Molinari et al.

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(1987) and Lebensohn and Tomé (1993) who proposed an alternative definition of the viscoplastic modulus based upon a tangent, rather than secant, formulation. Later, a first order affine linearization procedure for non linear elastic, viscoplastic and elastoplastic composites and polycrystals was developed by Masson et al. (2000). In particular, these authors showed that such a first order affine approximation implemented within a self-consistent framework improves the estimate of the effective behavior of non-linear polycrystals in comparison with the incremental formulation of Hill (1965).

The case of elasto-viscoplasticity raises additional difficulties due to the differential nature of the constitutive equations involving different orders of time derivation. Different possibilities have been explored to overcome these difficulties. In the case of linear viscoelasticity, Hashin (1969) and Laws and McLaughlin (1978) have used the correspondence principle to propose a hereditary approach. Indeed, the use of Laplace–Carson transforms allows separating time and space variables such that it is possible to define a single viscoelastic modulus in the Laplace-Carson space. The self-consistent problem is solved in the Laplace–Carson space before proceeding to the inversion to the real time space. Extensions to the non-linear case have been proposed by Rougier et al. (1994) and Masson and Zaoui (1999). The constitutive law is first linearized at each step of the deformation path. In this way, the considered problem is converted into a viscoelastic problem, which can be classically solved using the self-consistent or the Mori–Tanaka approximation in the Laplace–Carson space. While the theoretical foundations of this approach are robust, the numerical implementation is rather complex mostly because the inversion of Laplace–Carson transforms requires intensive computations. Recently, the results of an enhanced affine formulation for two-phase composites were compared to numerical results obtained by the finite element method in Pierard and Doghri (2006) and Pierard et al. (2007).

In contrast with hereditary formulations, internal variable approaches can be preferred for the simplicity of their numerical implementation. Indeed, the resolution of the heterogeneous problem is achieved in the real time space with some internal variables whose introduction in the strain and stress concentration relations allows remembering the material's history. The first developments were carried out by Weng (1981,) who adapted Kröner's model to the case of elasto-viscoplasticity. However, similarly to the original proposition of Kröner (1961), internal stresses are largely overestimated with this method (Zaoui and Raphanel, 1993). This approach was later extended to the case of finite strains by Nemat-Nasser and Obata (1986) and Harren (1991). Mercier and Molinari (2009) used the additive interaction law developed by Molinari et al. (1997) and Molinari (2002) to derive a self-consistent model based upon a tangent linearization of the viscoplastic flow rule. The additive interaction law has been later adapted to build a finite strain elasto-viscoplastic self-consistent model for polycrystals (Wang et al., 2010). Recently, an internal variable approach obtained from a variational method was developed by Lahellec and Suquet (2007) and Lahellec and Suquet (2007) to describe the behavior of composite materials. Paquin et al. (1999), Sabar et al. (2002) and Berbenni et al. (2004) proposed an alternative framework based upon the specific properties of projection operators. In the approach of Paquin et al. (1999), the purely elastic and purely viscoplastic heterogeneous problems are solved independently using the self-consistent approximation. Using translated field techniques, the individual solutions are combined to deduce a strain rate localization rule. Comparisons between both the translated field techniques and the additive interaction law were reported in Mercier et al. (2012) in the case of two-phase compressible or incompressible linear viscoelastic composites using a Mori–Tanaka approximation.

In the present paper, an extension of the translated field approach initiated by Paquin et al. (1999) for the modeling of disordered materials using the self-consistent approximation is proposed. The motivations are twofold. First, while the original approach of Paquin et al. (1999) derives from a classical secant approximation, this work aims at adopting an affine linearization procedure of the viscoplastic flow rule (i.e. a first order Taylor expansion of the viscoplastic flow rule) as suggested by Masson et al. (2000). Indeed, it has been observed that the first order affine formulation yields softer responses than the secant formulation (Masson and Zaoui, 1999; Masson et al., 2000; Molinari, 2002) which, in the case of pure viscoplasticity (or non linear elasticity in a general sense), may lead to the violation of a non-linear upper bound for the moduli (Gilormini, 1995). Second, in the original approach of Paquin et al. (1999), the local constitutive law only accounts for elastic and viscoplastic contributions. In the present work, the contribution of thermal expansion is also considered so that the proposed model may be used to investigate the response of disordered materials subjected to thermo-mechanical loading paths. Using the Mori–Tanaka approximation, a first order affine extension to the model of Paquin et al. (1999) has already been proposed for two-phase materials by Berbenni and Capolungo (2014). It was shown that predictions are very close to the ones obtained with the incremental variational approach developed by Lahellec and Suquet (2007) for two-phase fiber-reinforced composites. It is also thought that such translated field approaches are simpler to formulate than the recent incremental variational approaches for non-linear elastic-viscoplastic materials (Lahellec and Suquet, 2007; Brassard et al., 2012; Lahellec and Suquet, 2013) whose development within a self-consistent framework for polycrystalline materials has not been achieved yet.

The paper is structured as follows. In the first section, starting from field equations, the heterogeneous problem is written in the form of an integral equation. Then, the translated fields, which are obtained from the solutions of the purely thermo-elastic and purely viscoplastic heterogeneous problems, are introduced in order to simplify the integral equation. Finally, the localization rule associated with the proposed affine formulation is deduced from the application of the self-consistent method. As an illustration, some applications are presented in the second section. The case of two-phase fiber-reinforced composites is first examined in order to compare the results from the proposed self-consistent approach with those given by others (Paquin et al., 1999; Lahellec and Suquet, 2007; Berbenni and Capolungo, 2014). Then, the model is used to describe the behavior of polycrystalline materials when they are subjected to various thermomechanical loading conditions. To demonstrate the relevance of the proposed affine formulation, it is compared to the original classical secant formulation of Paquin et al. (1999) and to the full-field FFT spectral method of Moulinec and Suquet (1998).